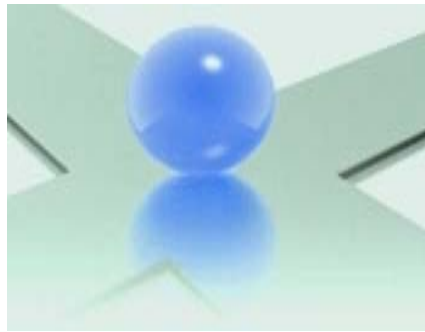


*Two* for the Price of *One*:  
Info-Gap Robustness  
of the  
One-Test Algorithm

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## Contents

<b>1</b>	<b>Highlights</b> (isipta2011-02poster.tex)	<b>3</b>
<b>2</b>	<b>Two Systems, One Test</b> (two-systems02poster.tex)	<b>4</b>
<b>3</b>	<b>Two Systems, One Test, CDF Known</b> (two-systems02poster.tex)	<b>5</b>
<b>4</b>	<b>Robustness of Two Systems, One Test</b> (two-systems02poster.tex)	<b>6</b>
<b>5</b>	<b>Three Systems, Two Tests</b> (two-systems02poster.tex)	<b>11</b>
<b>6</b>	<b>Three Systems, One Test</b> (two-systems02poster.tex)	<b>12</b>
<b>7</b>	<b><math>n</math> Systems, <math>m</math> Tests</b> (two-systems02poster.tex)	<b>12</b>
<b>8</b>	<b>Extensions</b> (isipta2011-02poster.tex)	<b>13</b>

# 1 *Highlights*

## § Binary decisions, expensive tests:

- **Airplane designs:**  
Lockheed or Raytheon?
- **Pollution control:**  
Tax or cap-and-trade?
- **Medical intervention:**  
Surgery or pharmaceuticals?
- **Military budget:**  
Tanks or intelligence?
- **Energy supply:**  
Nuclear or fossil?

## § 2 Systems, 1 Test: Probabilistic Alg.

## § Info-gap uncertainty on pdf: Robustify.

## § $n$ Systems, $m$ Tests.

## § Source: <http://info-gap.com>

## 2 *Two Systems, One Test*

§ **Two systems**, with qualities  $x_1 \neq x_2 \in \mathfrak{R}$ .

Bigger is better.

§ **No prior knowledge?**

- Flip a fair coin.
- 50/50 chance of success.

§ **One system tested:** quality  $x_r$ .

- Enhanced chance of success?
- Which system to use?

## § Algorithm for choosing a system:

- $q(y)$  is **any** pdf:  $q(y) > 0$  for all  $y$ .
- Draw  $y$  from  $q(y)$ .
- If  $y \geq x_r$  then **choose un-tested** system.
- If  $y < x_r$  then **choose tested** system.

## § Probability of success, $P_s(q)$ :

Probability of choosing larger  $x_i$ .

## § Theorem:

If tested system chosen with prob. 0.5,  
then  $P_s(q) > 0.5$ .

## 3 *Two Systems, One Test, CDF Known*

### § $F(x)$ is known cdf.

## § Algorithm for choosing a system:

- If  $F(x_r) < \frac{1}{2}$ , **choose un-tested** system.
- If  $F(x_r) \geq \frac{1}{2}$ , **choose tested** system.

## § Theorem: $P_s = \frac{3}{4}$

Proof: Robert R. Snapp, 2005.<sup>1</sup>

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<sup>1</sup>Robert R. Snapp, 2005, U of Vermont, \papers\2-systems-1test\isipta2011\covers-problem.pdf

## 4 *Robustness of Two Systems, One Test*

### § Recall no-knowledge algorithm:

- $q(y)$  is **any** pdf:  $q(y) > 0$  for all  $y \in \mathfrak{R}$ .
- Draw  $y$  from  $q(y)$ .
- If  $y \geq x_r$  then **choose un-tested** system.
- If  $y < x_r$  then **choose tested** system.

§ **Theorem:**  $P_s(q) > 0.5$ .

§ **How to choose  $q(y)$ ?**

Can we beat  $P_s(q) > 0.5$ ?

§ If we **know**  $p(x_i)$  then  $P_s = 0.75$ .

Can we achieve  $P_s(q) = 0.75$ ?

## § Info-gap robust-satisficing:

- **Our guess:**  $x \sim \tilde{p}(x)$ .
- $\tilde{p}(x)$  highly **uncertain**.
- **Choose  $q(y)$  to robust satisfy:**
  - **Satisfy**  $P_s \geq P_c$ .
  - **Maximize robustness** to uncertain  $\tilde{p}$ .

## § Info-gap model for uncertain $\tilde{p}(x)$ : $\mathcal{U}(h)$ .

- **Nesting:**  $h < h' \implies \mathcal{U}(h) \subseteq \mathcal{U}(h')$ .
- **Contraction:**  $\mathcal{U}(0) = \{\tilde{p}\}$ .
- $h$  is unbounded **horizon of uncertainty**.

## § Robustness, $\hat{h}(q, P_c)$ :

**Maximum tolerable uncertainty.**

$$\hat{h}(q, P_c) = \max \left\{ h : \left( \min_{p \in \mathcal{U}(h)} P_s(q|p) \right) \geq P_c \right\}$$

## § Example:

- **Estimated pdf:**  $\tilde{p}(x) = \tilde{\lambda}e^{-\tilde{\lambda}x}$ .
- **Decision pdf:**  $q(y) = \gamma e^{-\gamma y}$ .
- **Prob of success:**  $P_s(q|\tilde{p}) > 0.5$
- **Putative optimal choice:**

$$\begin{aligned}\gamma^* &= \arg \max_{\gamma} P_s(q|\tilde{p}) \\ &= \tilde{\lambda}\sqrt{2}\end{aligned}$$

- **E.g.,**  $\tilde{\lambda} = 1$ :  $P_s(q|\tilde{p}) = 0.67 \gg 0.5$
- **Robust to uncertainty in  $\tilde{p}(x)$ ???**



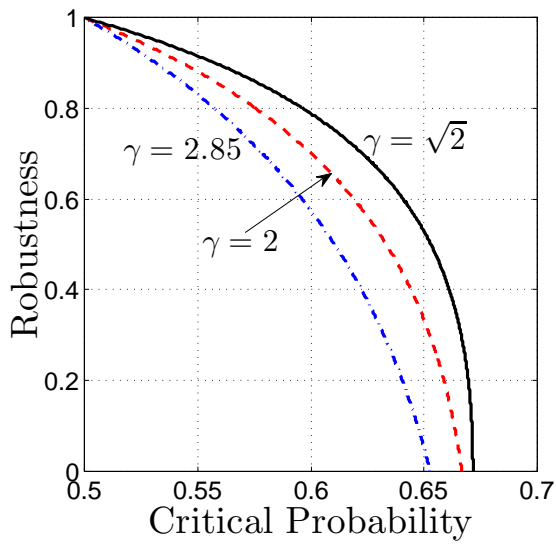


Figure 1: Robustness curves with  $\tilde{\lambda} = 1$ .

§ **Trade off:** robustness vs prob. of success.

§ **Zeroing:**

Estimated prob of success: no robustness.

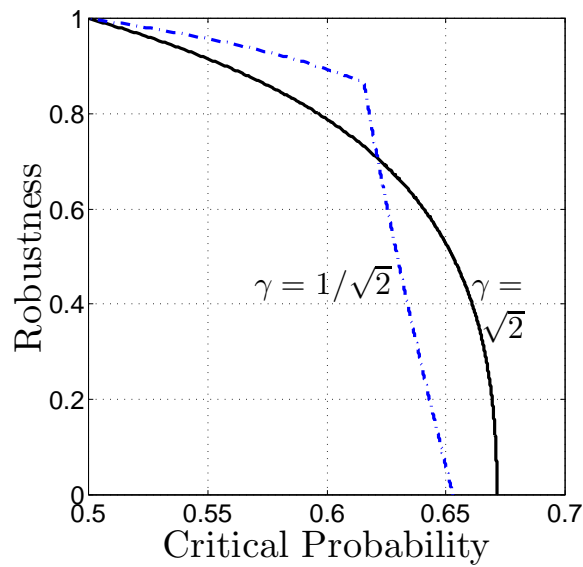


Figure 2: Robustness curves with  $\tilde{\lambda} = 1$ .

§ **Trade off:** robustness vs prob. of success.

§ **Zeroing:** no robustness of estimate.

§ **Preference reversal.**

- $\gamma = \sqrt{2}$  more robust for  $P_c > 0.62$ .
- $\gamma = 1/\sqrt{2}$  more robust for  $P_c < 0.62$ .

## 5 *Three Systems, Two Tests*

### § 3 Systems with qualities:

$$x_1 < x_2 < x_3$$

### § Test two systems with revealed attributes:

$$r_1 < r_2$$

### § Goal: Exclude worst system.

### § Blind probability of success: $\frac{1}{3}$

### § Algorithm:

- $q(y)$  **any** non-zero pdf.
- Draw  $y$  from  $q(q)$ .
- If  $y < r_1$  choose 2 tested systems.
- If  $r_1 \leq y$  choose  $r_2$  and untested system.

### § Theorem:

If tested systems chosen with equal prob.

then  $P_s(q) > \frac{1}{3}$ .

## 6 *Three Systems, One Test*

### § 3 Systems with qualities:

$$x_1 < x_2 < x_3$$

§ Test one system with revealed attribute  $r$ .

§ Goal: Select best system.

§ Blind probability of success:  $\frac{1}{3}$

§ Algorithm:

- $q(y)$  any non-zero pdf.
- Draw  $y$  from  $q(q)$ .
- If  $y \leq r$  choose tested system.
- If  $r < y$  choose equi-prob from untested.

§ Theorem:

If tested system chosen with equal prob.

then  $P_s(q) > \frac{1}{3}$ .

## 7 *n Systems, m Tests*

§ Hypothesized generalization to

$n$  systems,  $m$  tests.

## 8 *Extensions*

§ Multiple attributes.

§ Adaptive testing.

§ Best possible probability of success.

§ Robust satisficing.

§ Opportune windfalling.