Robust detection of exotic infectious diseases in animal herds: A comparative study of two decision methodologies under severe uncertainty

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Problem Description

Inspection Protocols

- animals are transported and pass through customs
- some of them may have dangerous infectious diseases
- how many animals to test, yet avoid cataclysmic events?

Model

extension of [2]

- allow for imperfect testing
- cost term for terminating the herd
- diseased animals modelled as binomial process
- worst-case assumption of independence between animals

Model Parameters

- n animals, of which m are to be tested
- d diseased animals (uncertain, see r)
- p test sensitivity, q test specificity
- r probability that a single animal is infected

$$n = 250$$
$$p = 0.9999$$
$$q = 0.999$$

Model Loss Function

- $c(m)$ cost of testing m animals
- $a(d)$ cost of d diseased animals passing inspection
- $t(n)$ cost of terminating n animals

$$c(m) = 1000 - 2000m + 1000m^2$$
$$a(d) = \begin{cases} 0 & \text{if } d = 0 \\ a & \text{if } d \geq 1 \end{cases} \qquad (a = 10\,000\,000)$$
$$t(n) = 400n = 100\,000$$

References

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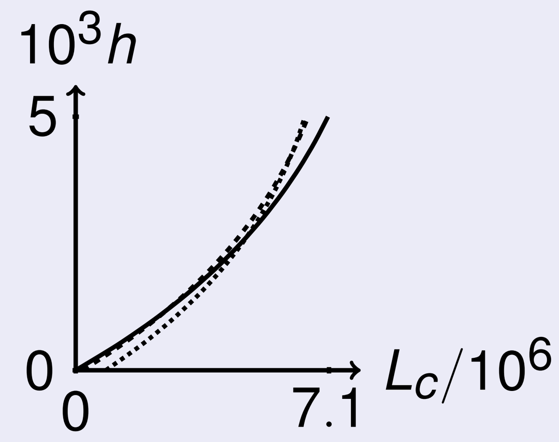
Comparison of Methodologies

Info-Gap Analysis [1]

select decision which meets a given performance criterion L_c under the largest possible range of r

$$\hat{h}(m, L_c) = \max_{h \geq 0} \left\{ h \colon \max_{r \in [0,h]} L(m|r) \leq L_c \right\}$$

robustness curve: $\hat{h}(m, L_c)$ as a function of L_c



$m = 1$ (solid), $m = 15$ (dashed), and $m = 30$ (dotted).

$L_c/10^6$	m^*	$10^3\hat{h}(m^*, L_c)$
0.5	2	0.207
1.5	5	0.661
2.5	8	1.184
3.5	11	1.803

Maximality [3]

- m dominates m' when $\min_{r \in [0,h]} \left[L(m'|r) - L(m|r) \right] > 0$
- pick one of the undominated options, i.e. any m for which

$$\min_{m' \in \{0,1,\ldots,n\}} \max_{r \in [0,h]} \left[L(m'|r) - L(m|r) \right] \geq 0$$

	$10^3 h$			
m	0.207	0.661	1.184	1.803
0	−0.9	−0.9	−0.9	−0.9
1	1.1	1.1	1.1	1.1
2	1.4	3.1	3.1	3.1
3	−0.6	4.9	5.1	5.1
4	−3.1	2.9	7.1	7.1
5	−7.7	0.9	7.0	9.1
6	−14.3	−1.1	5.0	11.1
7	−22.9	−4.3	2.9	9.9
8	−33.4	−9.5	0.9	7.9
9	−46.0	−16.6	−1.1	5.8
10	−60.6	−25.9	−4.3	3.7
11	−77.2	−37.1	−9.5	1.7
12	−95.8	−50.3	−16.8	−0.4
13	−116.4	−65.6	−26.1	−2.9
14	−139.1	−82.9	−37.4	−7.4
15	−163.7	−102.2	−50.8	−14.1

Discussion

- info-gap and maximality give essentially the same result
- this is not a coincidence!

Info-Gap–Maximin Theorem

The info-gap solution $D^*(L_c)$ coincides with Γ-minimax solution with respect to $\overline{P}_h$ whenever the following conditions are satisfied:

1. for all $d \in D$, $\overline{P}_h(L(d, \cdot))$ is strictly increasing as a function of h, and
2. it holds that $L_c = \min_{d \in D} \overline{P}_h(L(d, \cdot))$.

(so, 'free' to choose L_c under the additional assumption of continuity)

Info-Gap–Maximality Theorem

Let $L_c(h) = \min_{d \in D} \overline{P}_h(L(d, \cdot))$.
Then, for all $h' \leq h$, every info-gap decision $d^* \in D^*(L_c(h'))$ is maximal with respect to $\overline{P}_h$.

Final Remarks

- info-gap yields an elegant approach to capture maximal solutions
- info-gap gives rough idea of the size of the maximal set
- info-gap is a graphically appealing way to represent maximal solution
- robustness curves show trade-off between uncertainty and cost