Structural Reliability Assessment with Fuzzy Probabilities

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Abstract

The prediction of the behavior and reliability of engineering structures and systems is often plagued by uncertainty and imprecision caused by sparse data, poor measurements and linguistic information. Accounting for such limitations complicates the mathematical modeling required to obtain realistic results in engineering analyses. The framework of imprecise probabilities provides a mathematical basis to deal with these problems which involve both probabilistic and non-probabilistic sources of uncertainty. A common feature of the various concepts of imprecise probabilities is the consideration of an entire set of probabilistic models in one analysis. But there are differences between the concepts in the mathematical description of this set and in the theoretical connection to the probabilistic models involved. This study is focused on fuzzy probabilities, which combine a probabilistic characterization of variability with a fuzzy characterization of imprecision. We discuss how fuzzy modeling can allow a more nuanced approach than interval-based concepts. The application in engineering is demonstrated by means of two examples.

Keywords. Fuzzy Probabilities, Imprecise Probabilities, Failure Probability, Reliability Analysis.

1 Introduction

The analysis and reliability assessment of engineering structures and systems involves uncertainty and imprecision in parameters and models of different type. In order to derive predictions regarding structural behavior and reliability, it is crucial to represent the uncertainty and imprecision appropriately according to the underlying real-world information which is available. To capture variation of structural parameters, established probabilistic models and powerful simulation techniques are available for engineers, which are widely applicable to real-world problems; for example, see [24]. The required probabilistic modeling can be realized via classical mathematical statistics if data of a suitable quality are available to a sufficient extent.

In civil engineering practice, however, the available data are frequently quite limited and of poor quality. These limitations create epistemic uncertainty, which can sometimes be substantial. It is frequently argued that expert knowledge can compensate for the limitations through the use of Bayesian methods based on subjective probabilities. If a subjective perception regarding a probabilistic model exists and some data for a model update can be made available, a Bayesian approach can be very powerful, and meaningful results with maximal information content can be derived. Bayesian approaches have attracted increasing attention in the recent past and considerable advancements have been reported for the solution of various engineering problems [7, 15, 23]. An important feature of Bayesian updating is that the subjective influence in the model assumption decays quickly with growing amount of data. It is then reasonable practice to estimate probabilistic model parameters based on the posterior distribution, for example, as the expected value thereof.

When less information and experience are available, greater difficulties will be faced. If the available information is very scarce or is of an imprecise nature rather than of a stochastic nature, a subjective probabilistic model description may be quite arbitrary. For example, a distribution parameter may be known merely in the form of bounds. Any prior distribution which is limited to these bounds would then be an option for modeling. But the selection of a particular model would introduce unwarranted information that cannot be justified sufficiently. Even the assumption of a uniform distribution, which is commonly used in those cases, ascribes more information than is actually given by the bounds. This situation may become critical if no or only very limited data are available for a model update. The initial subjectivity is then dominant in the posterior distribution and in the final result. If these results, such as failure probabilities, determine critical decisions, one may wish to consider the problem from the following angle.

If several probabilistic models are plausible for the description of a problem, and no information is available to assess the suitability of the individual models or to relate their suitability with respect to one another, then it may be of interest to identify the worst case for the modeling rather than to average over all plausible model options with arbitrary weighting. The probabilistic analysis is carried out conditional on each of many particular probabilistic models out of the set of plausible models. In reliability assessment, this implies the calculation of an upper bound for the failure probability as the worst case. This perspective can be extended to explore the sensitivity of results with respect to the variety of plausible models, that is, with respect to a subjective model choice. A mathematical framework for an analysis of this type has been established with imprecise probabilities; see [28]. Applications to reliability analysis [17, 22, 26] and to sensitivity analysis [9, 13] have been reported. This intuitive view, however, is by far not the entire motivation for imprecise probabilities [16]. Imprecise probabilities are not limited to a consideration of imprecise distribution parameters. They are capable of dealing with imprecise conditions and dependencies between random variables and with imprecise structural parameters and model descriptions. Respective discussions can be reviewed, for example, in [8, 14]. Multivariate models can be constructed [11]. Imprecise probabilities also allow statistical estimations and tests with imprecise sample elements. Results from robust statistics in form of solution domains of statistical estimators can be considered directly and appropriately [1].

In this paper, the implementation of intervals and fuzzy sets as parameters of probabilistic models is discussed in the context of proposed concepts of imprecise probabilities. Structural reliability analysis is employed to illustrate the effects in examples.

2 Imprecise Probabilistic Model Parameters

In engineering analyses, parameters of probabilistic models are frequently limited in precision and are only known in a coarse manner. This situation can be approached with different mathematical concepts. First, the parameter can be considered as uncertain with random characteristics, which complies with the Bayesian approach. Subjective probability distributions for the parameters are updated by means of objective information in form of data. The result is a mix of objective and subjective information – both expressed with probability. Second, the parameter can be considered as imprecise but bounded within a certain domain, where the domain is described as a set. In this manner, only the limitation to some domain and no further specific characteristics are ascribed to the parameter, which introduces significantly less information in comparison with a distribution function as used in the Bayesian approach. Imprecision in the form of a set for a parameter does not migrate into probabilities, but it is reflected in the result as a set of probabilities which contains the true probability. Intervals and fuzzy sets can thus be considered as models for parameters of probability distributions.

An interval is an appropriate model in cases where only a possible range between crisp bounds x_l and x_r is known for the parameter x, and no additional information concerning value frequencies, preference, etc. between interval bounds is available nor any clues on how to specify such information. Interval modeling of a parameter of a probabilistic model connotes the consideration of a set of probabilistic models, which are captured by the set of parameter values

$$X_I = \left[x_l, x_r \right] \ . \tag{1}$$

This modeling corresponds to the p-box approach [10] and to the theory of interval probabilities [28, 29]. Events E_i are assessed with a range of probability, $\left[P_l\left(E_i\right), P_r\left(E_i\right) \right] \subseteq \left[0,1\right]$, which is directly used for the definition of interval probability, denoted as *IP*, as follows,

$$\begin{aligned} & IP: \quad E_\Omega \rightarrow I \quad with \\ & E_\Omega = \mathfrak{P}\left(\Omega\right),\ I = \left\{ \left[a,b\right]:\ 0 \le a \le b \le 1 \right\} \ . \end{aligned} \tag{2}$$

In Eq. (2), $\mathfrak{P}\left(\Omega\right)$ is the power set on the set Ω of elementary events ω. This definition complies with traditional probability theory. Kolmogorov's axioms and the generation scheme of events are retained as defined in traditional probability theory, see also [30]. Traditional mathematical statistics are applicable for quantification purposes. In reliability analysis with interval probabilities, the parameter interval X_I is mapped to an interval of the failure probability,

$$X_I \rightarrow P_{fI} = \left\{ P_f \,\middle|\, P_f \in \left[P_{fl}, P_{fr} \right] \right\} \ . \tag{3}$$

Scrutinizing the modeling of parameters as intervals shows that an interval is a quite crude expression of imprecision. The specification of an interval for a parameter implies that, although a number’s value is not known exactly, exact bounds on the number can be provided. This may be criticized because the specification of precise numbers is just transferred to the bounds. Fuzzy set theory provides a suitable basis for relaxing the need for precise values or bounds. It allows the specification of a smooth transition for elements from belonging to a set to not belonging to a set. Fuzzy numbers are a generalization and refinement of intervals for representing imprecise parameters. The essence of an approach using fuzzy numbers that distinguishes it from more traditional approaches is that it does not require the analyst to circumscribe the imprecision all in one fell swoop with finite characterizations having known bounds. The analyst can now express the available information in the form of a series of plausible intervals, the bounds of which may grow, including the case of infinite limits. This allows a more nuanced approach compared to interval modeling.

Fuzzy sets provide an extension to interval modeling that considers variants of interval models, in a nested fashion, in one analysis. A fuzzy set $\tilde{X}$ of parameter values can be represented as a set of intervals X_I,

$$\tilde{X} = \left\{ \left(X_\alpha, \mu\left(x \in X_\alpha\right)\right) \middle| \begin{array}{l} X_\alpha = X_I, \\ \mu\left(x \in X_\alpha\right) = \alpha \\ \forall \alpha \in (0,1] \end{array} \right\} . \tag{4}$$

This is utilized for an approximation of $\tilde{X}$ via a series of discrete values $\alpha_i \in (0,1]$, which is referred to as α-discretization; see Figure 1 [31]. In Eq. (4), X_α denotes an α-level set of the fuzzy set $\tilde{X}$, and $\mu(.)$ is the membership function. This modeling applied to parameters of a probabilistic model corresponds to the theory of fuzzy random variables and to fuzzy probability theory according to [4, 18]. For further information on related concepts, see [6, 12, 19]. The definition of a fuzzy random variable refers to imprecise observations as outcome of a random experiment. A fuzzy random variable $\tilde{\mathrm{Y}}$ is the mapping

$$\tilde{\mathrm{Y}}:\ \Omega \rightarrow \mathfrak{F}\left(\mathbf{Y}\right) \tag{5}$$

with $\mathfrak{F}(\mathbf{Y})$ being the set of all fuzzy sets on the fundamental set $\mathbf{Y}$, whereby the standard case is $\mathbf{Y} = \mathrm{R}^n$. The pre-images of the imprecise events described by $\mathfrak{F}(\mathbf{Y})$ are elements of a traditional probability space $[\Omega, \mathfrak{S}, P]$. This complies with traditional probability theory and allows statistics with imprecise data [2, 18, 27]. As a consequence of Eq. (5), parameters of probabilistic models, including descriptions of the dependencies and distribution type, and probabilities are obtained as fuzzy sets. This builds the relationship to the p-box approach and to the theory of interval probabilities. A representation of a fuzzy probability distribution function of a fuzzy random variable $\tilde{\mathrm{Y}}$ with aid of α-discretization leads to interval probabilities $\left[F_{\alpha l}\left(y\right), F_{\alpha r}\left(y\right)\right]$ for each α-level as one plausible model variant,

$$\tilde{F}\left(y\right) = \left\{\left(F_\alpha\left(y\right), \mu\left(F\left(y\right) \in F_\alpha\left(y\right)\right)\right)\ \right\} \tag{6}$$

with

$$F_\alpha\left(y\right) = \left[F_{\alpha l}\left(y\right), F_{\alpha r}\left(y\right)\right]\ , \tag{7}$$

$$\mu\left(F\left(y\right) \in F_\alpha\left(y\right)\right) = \alpha \forall \alpha \in (0,1]\ . \tag{8}$$

As depicted in Figure 1, in a reliability analysis, the fuzzy set $\tilde{X}$ of parameter values is mapped to a fuzzy set of the failure probability,

$$\tilde{X} \rightarrow \tilde{P}_f\ . \tag{9}$$

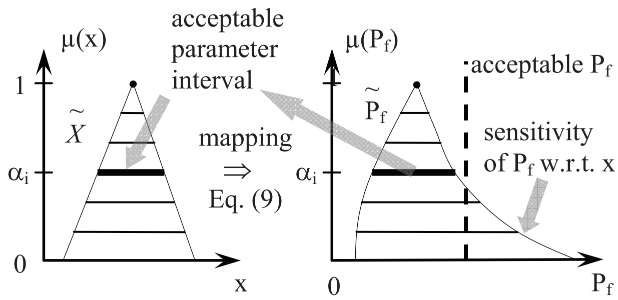

Figure 1: Relationship between fuzzy parameters and failure probability.

The membership function serves only instrumentally to summarize various plausible interval models in one embracing scheme. The interpretation of the membership value μ as epistemic possibility, which is sometimes proposed, may be useful for ranking purposes, but not for making critical decisions. The importance of fuzzy modeling lies in the simultaneous consideration of various magnitudes of imprecision at once in the same analysis.

The features of a fuzzy probabilistic analysis can be utilized to identify sensitivities of the failure probability with respect to the imprecision in the probabilistic model specification; see Figure 1. Sensitivities of P_f are indicated when the interval size of $P_{f\alpha}$ grows strongly with a moderate increase of the interval size of X_α of the parameters. If this is the case, the membership function of $\tilde{P}_f$ shows outreaching or long and flat tails. An engineering consequence would be to pay particular attention to those model options X_α, which cause large intervals $P_{f\alpha}$ and to further investigate to verify the reasoning for these options and to possibly exclude these critical cases.

A fuzzy probabilistic analysis also provides interesting features for design purposes. The analysis can be performed with coarse specifications for design parameters and for probabilistic model parameters. From the results of this analysis, acceptable intervals for both design parameters and probabilistic model parameters can be determined directly without a repetition of the analysis; see Figure 1. Indications are provided in a quantitative manner to collect additional specific information or to apply certain design measures to reduce the input imprecision to an acceptable magnitude. This implies a limitation of imprecision to only those acceptable magnitudes and so also caters for an optimum economic effort. For example, a minimum sample size or a minimum measurement quality associated with the acceptable magnitude of imprecision can be directly identified. Further, revealed sensitivities may be taken as a trigger to change the design of the system under consideration to make it more robust. A related method is described in [5] for designing robust structures in a pure fuzzy environment. These methods can also be used for the analysis of aged and damaged structures to generate a rough first picture of the structural integrity and to indicate further detailed investigations to an economically reasonable extent–expressed in form of an acceptable magnitude of input imprecision according to some α-level.

3 Examples

3.1 Concept Demonstration: Reinforced Concrete Frame

The principle of the fuzzy probabilistic reliability analysis is illustrated by means of the reinforced concrete frame from [22] shown in Figure 2.

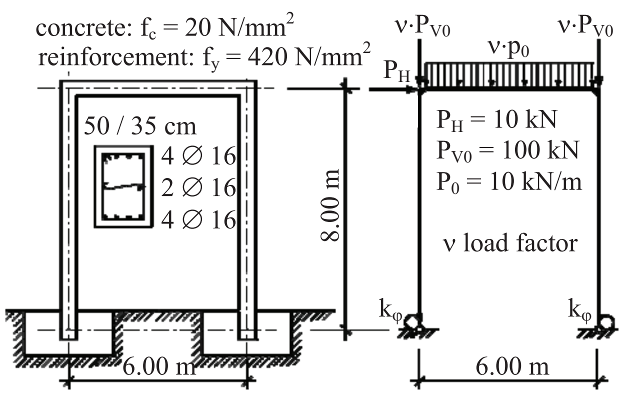

Figure 2: Reinforced concrete frame, structural model, and loading.

The structure is loaded by its dead weight, a small horizontal load P_H, and the vertical loads P_{V0} and p_0 which are increased with the factor ν until global structural failure is reached. For the purpose of demonstration, only the load factor ν is introduced as a random variable with an extreme value distribution of Ex-Max Type I (Gumbel) with mean $\tilde{m}_\nu$ and standard deviation $\tilde{\sigma}_\nu$. Imprecision of the probabilistic model is described with triangular fuzzy numbers $\tilde{m}_\nu = \langle 5.7, 5.9, 6.0 \rangle$ and $\tilde{\sigma}_\nu = \langle 0.08, 0.11, 0.12 \rangle$. In addition, the rotational stiffness of the springs at the column bases is modeled as a triangular fuzzy number $\tilde{k}_\varphi = \langle 5, 9, 13 \rangle$ MNm/rad to take account of the only vaguely known soil properties. These fuzzy parameters are considered as given for the purpose of this paper to highlight certain advantages of fuzzy probabilistic approaches in structural reliability assessment rather than to demonstrate the procedure for a specific practical case. In practical applications these fuzzy parameters need to be determined for the specific case. Although a general rule or algorithm cannot be formulated for this purpose, expert knowledge and inspection results are frequently available, which can be used together with statistical methods to determine bounds for the support of these parameters in a conservative manner. These semi-heuristic approaches can then be extended to higher α-levels in order to derive further nested intervals with an engineering meaning, e.g., to which the parameter imprecision can be reduced with certain technical efforts. Some suggestions to derive fuzzy parameters of probability distributions based on statistical data with typical characteristics as in civil engineering practice are discussed in [3]. It should be noted that the membership values are only instrumental in this approach with no specific meaning; they enable the simultaneous consideration of a variety of intervals of different size at once in the same analysis; see Section 2.

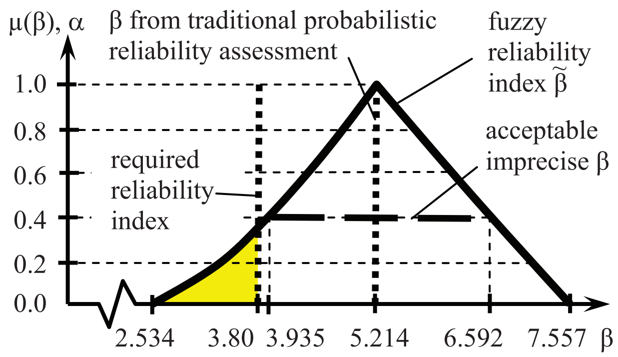

Figure 3: Fuzzy reliability index and evaluation against safety requirement.

Based on this input information, the fuzzy reliability index $\tilde{\beta}$ shown in Figure 3 is calculated. The result spreads over a large range of possible values for β. The interval bounds for each α-level are determined with the global optimization approach from [21], which is based on a modified evolution strategy. This provides advantages over a perturbation method or sensitivity investigation in view of result accuracy as the dependency between the parameters and β can be quite nonlinear, and the intervals obtained for β are quite large. The shaded part of $\tilde{\beta}$ does not comply with the safety requirements. This means that a sufficient structural reliability is not ensured when the parameters are limited to the plausible ranges for α = 0. In a traditional reliability analysis, using crisp assumptions for the parameters out of their plausible range such as the values associated with the membership μ = 1, this critical situation is not revealed. So far, the results from p-box approach or from interval probabilities would lead to the same conclusions. As an additional feature of fuzzy probabilities, it can be observed that the left tail of the membership function of $\tilde{\beta}$ slightly tends to flatten towards small values. This indicates a slight sensitivity of β with respect to imprecision of the fuzzy input when this grows in magnitude. So one may wish to reduce the input imprecision to a magnitude which is associated with the steeper part of the membership function of β. In Figure 3, the part $\mu(\beta) \geq 0.4$ is a reasonable choice in this regard. Further, the result $\beta_{\alpha=0.4}$ = [3.935, 6.592] for $\mu(\beta) \geq 0.4 = \alpha$ (according to the definition of α-level sets) satisfies the safety requirement $\beta_{\alpha=0.4} \geq 3.8$. That is, a reduction of the imprecision of the fuzzy input parameters to the magnitude on α-level α = 0.4 would lead to an acceptable reliability of the structure despite the remaining imprecision in the input. For example, a collection of additional information can be pursued to achieve the requirements

- $k_\varphi \in$ [6.6, 11.4] MNm/rad = $k_{\varphi,\alpha=0.4}$,
- $m_\nu \in [5.78, 5.96] = m_{\nu,\alpha=0.4}$,
- $\sigma_\nu \in [0.092, 0.116] = \sigma_{\nu,\alpha=0.4}$.

If this cannot be achieved for one or more parameters, the fuzzy analysis can be repeated with intervals for the parameters with non-reducible imprecision and with fuzzy sets for the parameters with reducible imprecision to separate the effects. The evaluation of the results then leads to a solution with proposed reduction of the imprecision only of those parameters for which this is possible. In this manner, it is also possible to explore sensitivities of the result β with respect to the imprecision of certain groups of input parameters or of individual input parameters. The repetition of the fuzzy analysis for these purposes can be avoided largely when a global optimization technique is used for the fuzzy analysis. This type of fuzzy analysis leads to a set of points distributed over the value ranges of the fuzzy input parameters and associated with results $\beta \in \tilde{\beta}$. For each construction of membership functions for the fuzzy input parameters, it is then immediately known which points belong to which α-level so that a discrete approximation of a result can be obtained directly without a repeated analysis. Repetition of the analysis is then only required for a detailed verification.

3.2 Practical Application: Offshore Structures

Reliability analysis of existing offshore structures in seawater conditions requires realistic models for corrosion. Due to scarce and imprecise information, however, the model parameters cannot be specified precisely and are merely known in form of bounds. This situation can be approached appropriately with concepts of imprecise probabilities.

3.2.1 Corrosion Model

A probabilistic model for mild steel corrosion based on results from various coupon tests and other observations is proposed in [20]. This model describes the material loss due to corrosion as a function of time; see Figure 4.

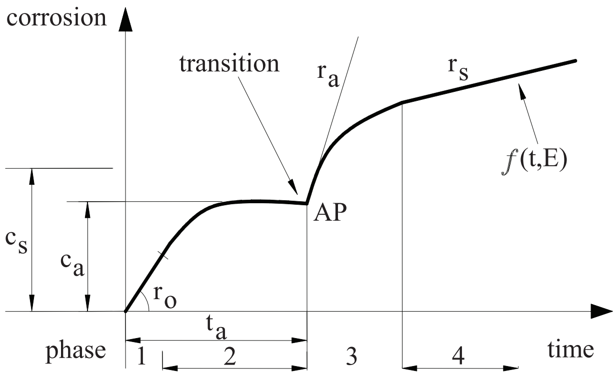

Figure 4: Corrosion model with mean value function $f(t,E)$ after [20].

Uncertainties in the corrosion process are considered with a probabilistic model for the corrosion depth $c(t,E)$, measured in mm, as

$$c(t,E) = b(t,E) \cdot f(t,E) + \varepsilon(t,E) \ , \tag{10}$$

with

- $f(t,E)$ – mean-value function,
- $b(t,E)$ – bias function,
- $\varepsilon(t,E)$ – zero-mean uncertainty function,
- E – vector of environmental (and material) parameters.

The specification of the mean-value function $f(t,E)$ requires calibration of the parameters shown in Figure 4. These parameters can be determined as a function $F(T)$ of the average seawater temperature T (contained in E),

$$\{r_0, t_a, c_a, r_a, c_s, r_s\} = F(T) \ , \tag{11}$$

see [20]. The variability of $c(t,E)$ is modeled with the zero-mean uncertainty function $\varepsilon(t,E)$ (in Eq. (10)) in the form of Gaussian white noise; $\varepsilon(t,E)$ is assumed with zero mean and a standard deviation given by

$$\sigma_\varepsilon(t,T) = (0.006 + 0.0003 \cdot T) \cdot \frac{t}{t_a} \ , \ \frac{t}{t_a} \leq 1.5 \ . \tag{12}$$

The bias function $b(t,E)$ in Eq. (10) reflects the difference of the mean value predicted by the corrosion model and the mean values of corrosion loss derived from data. It is a function of the exposure time. Examples for bias functions based on statistical evaluations are provided in [20], see Figure 5, as functions of the non-dimensional time coordinate t/t_a with t_a as shown in Figure 4.

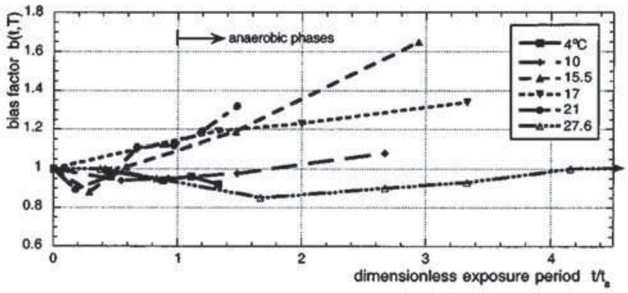

Figure 5: Bias function $b(t,E)$, dimensionless, after [20].

Before the anaerobic phases (up to the end of phase 2), the bias function lies in the range between 0.9 and 1.1. In the anaerobic phases (phases 3 and 4), the spread between the possible graphs becomes even more distinctive. A dependency between the temperature T and bias function $b(t,E)$ cannot be retrieved based on this information only. A condensation of the spread into a deterministic bias function would disregard information. On the other hand, the available information on the spread is quite sparse for the specification of a probabilistic model with sufficient confidence. A Bayesian approach would require some data for model update. If this is not available, as can be assumed for this type of data for a specific location, the model would remain subjective. Thus, one may wish to identify the worst case for the bias function $b(t,E)$ for the analysis based on the range of available information. But a simple conclusion such as “the upper bound of the bias function leads to the most critical structural behavior” may not apply. Due to the variety of members in a structural system even a uniform thickness reduction can lead to changes in kinematic failure modes. This motivates a search for the worst case under consideration of a plausible range for the bias function $b(t,E)$.

In the subsequent two examples, the uncertainty of the bias function $b(t,E)$ is accounted for with different models, and the effects on the results of a corresponding reliability analysis are investigated.

3.2.2 Steel Plate

For demonstration purposes, an example of a simple steel plate is taken from [20], and a reliability assessment is carried out under uncertain corrosion impact. The effects of different models for the uncertainty of the bias function $b(t,E)$ are investigated with respect to the failure probability P_f. The analysis is limited to the aerobic corrosion phase. It is assumed that the steel plate is exposed to seawater with a temperature of $T = 15°C$ over a period of 2.5 years.

Let d and h denote the thickness and nominal width of the uniform plate, respectively. A load is applied to cause a constant uniaxial tensile force Q in the plate. The force Q follows a normal distribution with parameters given in Table 1. It is applied at $t = 2.5$ years.

Variable	Mean	Standard deviation
Q	200 kN	23 kN
S_y	300 MPa	10 MPa
d	4 mm	0
h	250 mm	0

Table 1: Example data summary.

The resistance $R(t)$ of the plate is expressed in terms of the yield stress S_y, and the cross sectional area is reduced by the corrosion loss $c(t,E)$ on both surfaces of the plate. That is,

$$R(t) = S_y \cdot h \cdot \left[d - 2 \cdot c(t,T) \right] . \tag{13}$$

The yield stress S_y is modeled as normally distributed. The performance function is

$$G(t) = R(t) - Q . \tag{14}$$

The corrosion model is specified according to [20], which leads to a mean value $f(.) = 0.3$ mm and to a standard deviation $\sigma_\varepsilon = 0.0126$ mm for the considered $t = 2.5$ years.

The failure probability P_f is first computed with a deterministic value for the bias function, $b_{det}(.) = 1.0$. Direct Monte Carlo simulation (MCS) with a sample size of $N_{Pf} = 10^5$ leads to $P_{f,det} = 0.0126$.

The bias factor $b(.)$ is considered as merely known lying in the range between 0.9 and 1.1, which represents model uncertainty. This complies with the information provided in Figure 5. For a purely probabilistic analysis, this range is taken into account with the aid of bounded random quantities. A common probabilistic model used for those purposes in engineering is the Beta distribution with its probability density function (pdf)

$$f_X(x) = \frac{1}{B(q,r)} \frac{(x-a)^{q-1}(b-x)^{r-1}}{(b-a)^{q+r-1}} \tag{15}$$

where $B(q, r)$ is the Beta function, and the parameters a and b are the minimum and maximum value of the random variable X, respectively, with $a \le x \le b$. This model can be adjusted quite arbitrarily by means of the distribution parameters. As the available information for the modeling of the bias $b(.)$ is quite scarce, possible variants for the distribution function for $b(.)$ are considered. The following cases of parameter adjustments are investigated: Case (I): $q = r = 1$, Case (II): $q = r = 2$, and

Case (III): $q = r = 3$; see Figure 6. Case (I) represents a uniform distribution, which is frequently used when no information about the distribution is available.

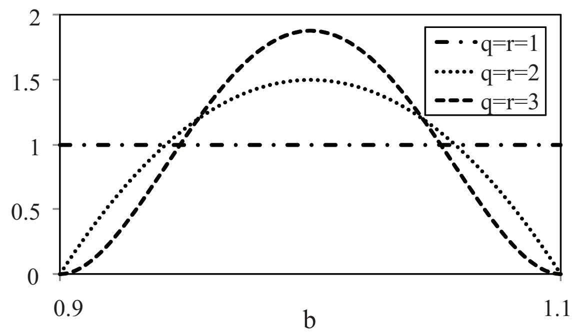

Figure 6: Variants for the pdf of the beta distribution.

The results of the subsequent reliability analysis provide extended information in comparison to the deterministic value $P_{f,det}$. To show the effects of the subjective distribution assumption on the result for P_f, a distribution for P_f is determined as dependent on the distribution of $b(.)$. An MCS is carried out for each sampling point $b(.)$ to obtain a corresponding value of $P_f(b)$, and the empirical distribution for P_f is constructed based on a sample size of $N_b = 2000$. The sample size for the determination of P_f for a given $b(.)$ is fixed at $N_{Pf} = 10^5$. The resulting plot of the distributions for the failure probability P_f in Figure 7 shows the differences between the cases considered. Since all cases represent possible models, their differences will be manifested through the distribution of P_f and their corresponding expectations $E[P_f]$ estimated in Figure 7.

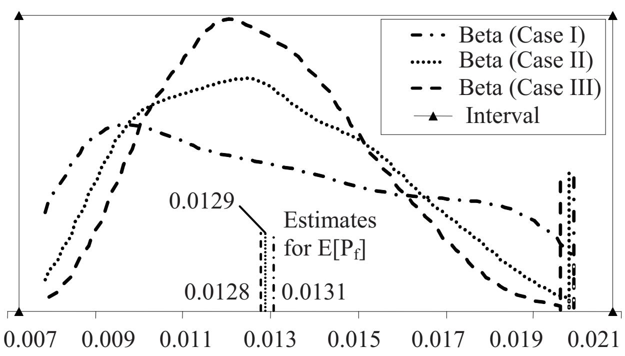

Figure 7: Failure probability, pdf's, means and upper bounds.

The modeling of $b(.)$ as a random variable involved data for various conditions and presumed variation of $b(.)$, which is reasonable for an analysis in a general context. For an analysis for a specific location, for which no data are available, one may wish to follow another approach. The bias function $b(.)$ may then be considered as given but unknown instead of showing variation. From this point of view, it is reasonable to determine the upper bound of P_f. With the stochastic parameter model, the upper bound for P_f can easily be retrieved from the sampling results shown in Figure 7, when the sampling is done conditional on $b(.)$. The results for the upper bounds in the considered cases are:

- Case (I): $P^u_{f,(I)}\left(b(.)\right) = 0.0199$,
- Case (II): $P^u_{f,(II)}\left(b(.)\right) = 0.0198$,
- Case (III): $P^u_{f,(III)}\left(b(.)\right) = 0.0196$.

The differences between these results for all three cases are quite small. The absolute values, however, are smaller than the true upper bound $P^u_{f,true}\left(b(.)\right) = 0.02082$. An improvement can be obtained by increasing the sample size N_b for $b(.)$. But a reasonable precision of $P^u_f\left(b(.)\right)$ demands a quite high numerical effort; the total number of evaluations of the limit state function is $N_b \cdot N_{Pf}$. This is hardly feasible for real structures, even when sophisticated sampling schemes are implemented.

Certainly, in a number of practical cases, including this simple example, the worst case for the imprecise parameter can be recognized in advance, so that the upper bound of P_f can be found easily. However, in a general case when the dependency between imprecise model parameters and P_f is non-monotonic, the solution is quite tedious.

A suitable approach to solve this problem is available with concepts of imprecise probabilities. The bias function is now modeled as an interval, $b_I = [0.9,1.1]$. An interval analysis is performed to map b_I to an interval for the failure probability $P_{f\ I} = \left[P^l_f\left(b(.)\right), P^u_f\left(b(.)\right)\right]$, see Eq. (3). The associated result is shown in Figure 7. This analysis is realized with the global optimization algorithm from [21]. Instead of sampling $b(.)$, a search algorithm is used to directly head for the interval bounds $P^l_f\left(b(.)\right)$ and $P^u_f\left(b(.)\right)$. Still, for each selected value $b(.) \in b_I$ an MCS needs to be carried out. The required number N_b of these simulations, however, is now significantly smaller; the exact result of the upper bound $P^u_f\left(b(.)\right)$ is approached much faster. With standard adjustments for the search algorithm, only $N_b = 45$ values of $P_f\left(b(.)\right)$ were calculated to find the true result $P^u_{f,true}\left(b(.)\right) = 0.02082$. This effort can be reduced further with an improved adjustment in the parameters of the search algorithm. The effort increases almost linearly with the number of interval input variables.

This analysis can be extended further by implementing a fuzzy probabilistic concept. This enables modeling of the bias function $b(.)$ with the aid of fuzzy sets so that a set of different intervals for $b(.)$ can be considered simultaneously. A rational approach is to assign a membership value $\mu(b(.)) = 1.0$ to the deterministic value $b_{det}(.) = 1.0$. A reasonable interval $b_{I0}(.) = [b_0^l(.), b_0^u(.)]$ may then be specified, which is even larger than the one concluded from available information, in order to reveal effects in case that $b(.)$ takes on exceptional values. The associated membership values are assigned as $\mu(b_0^l(.)) = \mu(b_0^u(.)) = 0.0$. In the example, $b_{I0} = [0.8,1.2]$ is selected. If no further specifications for membership values are made, this leads to the fuzzy triangular number $\tilde{b}(.) = \langle 0.8,1.0,1.2 \rangle$ as shown in Figure 8. Of course, the interval concluded from available information should be included in the fuzzy modeling. This is provided in form of the α-level set $b_{I\alpha}(.) = b_I(.) = [0.9,1.1]$ for $\alpha = \mu(b(.)) = 0.5$; see Figure 8. The associated analysis is performed with global optimization according to [21] as a repetition of the interval analysis for various membership levels with exploitation of the nested configuration of the intervals. A fuzzy failure probability $\tilde{P}_f$ is obtained as shown in Figure 8.

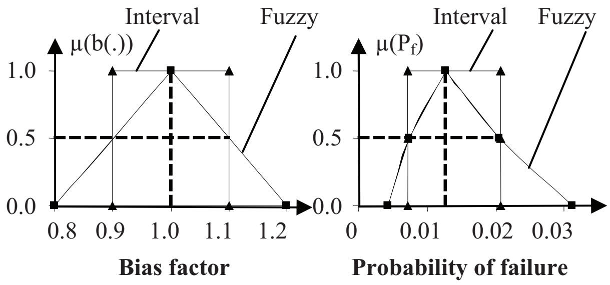

Figure 8: Fuzzy bias factor $\tilde{b}(.)$ and fuzzy failure probability $\tilde{P}_f$; interval modeling and results from Figure 7 are included for $\mu = \alpha = 0.5$.

A total of $N_b = 208$ calculations of $P_f(b(.))$ were necessary to obtain this result. The number N_b in the fuzzy analysis is not a multiple of N_b from interval analysis according to the number of α-levels. Random elements in the optimization procedure weaken this conclusion to the statistical mean of N_b. The search domains for different α-levels are of a different and so require a different N_b. Further, the numerical procedure from [21] exploits the nested configuration of the interval to re-use all previously evaluated points inside the search domain, which leads to a significant gain in numerical efficiency for a larger number of α-levels. In the example, the significant increase of the support of the parameters in the fuzzy analysis compared to the interval possesses the governing effect, which leads to increase of N_b by a factor larger than two. But this is still a much smaller number N_b compared to a stochastic sampling of $b(.)$. Compared to interval analysis, the numerical effort is higher. But the result $\tilde{P}_f$ is much richer in information compared to P_{fI}. The fuzzy analysis contains the above interval analysis on the level $\alpha = 0.5$; see Figure 8. In addition, a series of intervals with decreasing and increasing size are analyzed, which provides information regarding sensitivities of P_{fI} with respect to the interval size of $b_I(.)$ as discussed in Sections 2 and 3.1. Again, the membership values are not of interest, they just serve as a tool in the modeling. Dependencies between the size of $b_I(.)$ and the size of P_{fI} become directly visible in the results. In the example, no particular sensitivities are obvious.

3.2.3 Offshore Platform

Deterioration of structural strength is a major factor in the safety assessment of offshore structures. The protective paints and cathodic protection may be ineffective after some years. Typically, when analyzing structural strength or structural capacity, only "uniform" corrosion is considered [20]. These issues can be addressed in an investigation as demonstrated in Section 3.2.2 applied to real structures. In the following example, a fixed offshore platform is analyzed, which is exposed to seawater with a temperature of $T = 15°C$ over a period of 5 years. All the tubular structural members beneath the seawater surface are assumed to have the same average reduction in thickness due to corrosion only on the outer side.

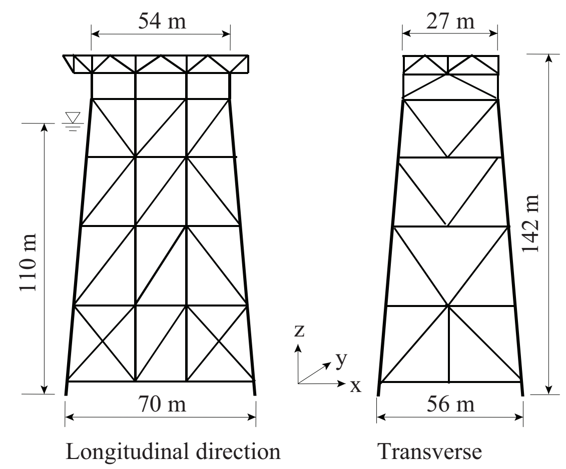

Figure 9: Structural model of the fixed jacket platform.

As an example structure, a fixed jacket platform located in the North Sea is taken from [25]. The jacket is designed for a water depth of approximately 110 m. The 8-leg jacket is arranged in a two by four rectangular grid. The overall dimensions are 27 m × 54 m at the top elevation and 56 m × 70 m at the mudline. The total height is 142 m. Horizontal bracings are installed at 5 levels. The jacket foundation consists of four corner clusters with eight skirt piles in each group and no leg piles are used. The longitudinal jacket frames are diagonal-braced, with X-braces between central and corner legs at the bottom bay. Transverse frames are K-braced, with the bottom K inverted to form a double X as shown in Figure 9.

The reliability analysis of a jacket structure involves the performance function,

$$G = \textit{Ultimate Resistance} - \textit{Environmental Loads}. \quad (16)$$

The ultimate resistance is determined through a pushover analysis of the platform. It is equal to the environmental design loads multiplied by the Reserve Strength Ratio (RSR). For this example, the environmental design loads are a 100-year wave together with a 10-year current. This is associated with a Gumbel distribution, which is implemented as a probabilistic load model in the analysis. For the structural resistance, uncertainty is considered in the yield strength of the steel and in the thickness reduction of the members due to marine corrosion. The yield strength of the steel ASTM-A7 is described with a log-normal distribution. Based on the probabilistic corrosion model discussed in Section 3.2.2, the environmental condition with $T = 15°C$ and $t = 5$ years leads to the mean value $f(.) = 0.48$ mm and the standard deviation $\sigma_\varepsilon = 0.08$ mm. The bias factor $b(t,T)$ lies in the range between 0.8 and 1.6 based on Figure 5. Implementation of these models in a structural analysis leads to the approximate performance function

$$G = \left[0.0704 \cdot F_y - 0.0887 \cdot c(t,T) - 0.0605\right] \cdot L_{100} - 0.0176 \cdot H^{2.2} \quad (17)$$

with

$$L_{100} = 0.0176 \cdot H_{100}^{2.2} . \quad (18)$$

For the reliability analysis, the variables in Eq. (17) are described by their respective probabilistic models. These random variables are summarized in Table 2. The probability of failure is calculated as $P_f = P(G \le 0)$ via MSC. In order to calculate P_f effciently, importance sampling is utilized. A sample size of $N_{Pf} = 5000$ is used for the reliability analysis. Variants for modeling of $b(.)$ are investigated, and the results are summarized in Figure 10. Again, the interval concept shows some advantage when the bounds on the failure probability have to be found. The total number of calculations N_b of P_f using the interval concepts is 114. The accuracy of the upper bound on P_f is higher, compared to the sampling of $b(.)$.

Variable	Distribution	Parameters	
F_y	Log. Normal	$\mu = 40\ psi$	$c.o.v. = 0.087$
H	Gumbel	$\alpha_H = 21.0\ m$	$\beta_H = 1.63\ m$
$c(.)$	Normal	$\mu = 0.48\ mm$	$\sigma_\varepsilon = 0.08\ mm$

Table 2. Random variables for the reliability analysis.

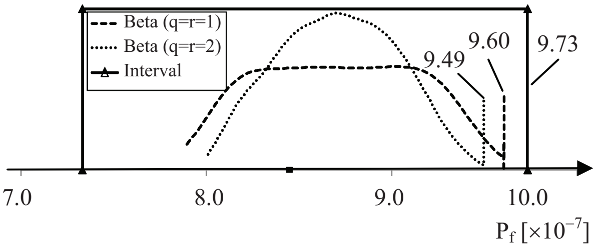

Figure 10: Failure probability; distributions, upper bounds and interval solution.

In the example, the differences in the upper bound on the failure probability are small. However, in other cases, and if more imprecision is involved in the problem, the discussed effects may become quite significant. It is obvious that the imprecision in the bias function $b(.)$ and thus, the imprecision of P_f grow dramatically with the exposure time, as can be seen in Figure 5. Further, in the example, only the annual failure probability is calculated. In a consideration of the failure probability for the entire lifetime of the structure, the imprecision in the annual failure probabilities will be accumulated accordingly. A consideration of this imprecision in a reliability analysis for the entire lifetime of an offshore structure is thus of great interest.

4 Summary and Conclusions

Different approaches were applied to describe imprecision in probabilistic models for a reliability analysis of engineering structures. The features of the models were compared with a pure probabilistic solution and with one another by means of academic and practical examples. The influence of the modeling on the prediction of structural reliability was examined. It was found that concepts of imprecise probabilities and, in particular, fuzzy probabilities, have certain advantages when bounds on the failure probability are of interest. These advantages concern the precision and the numerical effort in the calculation of these bounds and, in the case of fuzzy probabilities, some extended insight into sensitivities of the computational results with respect to the imprecision of the probabilistic input. Applicability in practice was demonstrated by means of a reliability analysis for a real offshore platform.

References

[1] Augustin, Th. and Hable, R. (2010). On the impact of robust statistics on imprecise probability models: a review, *Structural Safety*, 32(6): 358–365.

[2] Bandemer, H. and Näther, W. (1992). *Fuzzy Data Analysis*, Kluwer Academic Publishers, Dordrecht.

[3] Beer, M. (2009). Engineering Quantification of Inconsistent Information, *International Journal of Reliability and Safety*, 3(1–3), 174–200.

[4] Beer, M. (2009). Fuzzy probability theory, in: R. Meyers (Ed.), *Encyclopedia of Complexity and Systems Science*, Vol 6, Springer, New York, 4047–4059.

[5] Beer, M. and Liebscher, M. (2008). Designing robust structures – a nonlinear simulation based approach, *Computers and Structures*, 86(10), 1102–1122.

[6] Couso, I. and Dubois, D. (2009). On the variability of the concept of variance for fuzzy random variables, *IEEE Transactions on Fuzzy Systems*, 17, 1070–1080.

[7] Der Kiureghian, A. and Ditlevsen, O. (2009). Aleatory or epistemic? Does it matter?, *Structural Safety*, 31(2), 105–112.

[8] Fellin, W., Lessmann, H., Oberguggenberger, M. and Vieider, R. (eds.) (2005). *Analyzing Uncertainty in Civil Engineering*, Springer, Berlin Heidelberg New York.

[9] Ferson, S. and Tucker, W. T. (2006). Sensitivity analysis using probability bounding, *Reliability Engineering & System Safety*, 91(10–11), 1435–1442.

[10] Ferson, S. and Hajagos, J. G. (2004). Arithmetic with uncertain numbers: rigorous and (often) best possible answers, *Reliability Engineering & System Safety*, 85(1–3), 135–152.

[11] Fetz, Th. and Oberguggenberger, M. (2010). Multivariate models of uncertainty: A local random set approach, *Structural Safety*, 32(6): 417–424.

[12] Gil, M. A., López-Díaz, M. and Ralescu, D. A. (2006). Overview on the development of fuzzy random variables, *Fuzzy Sets and Systems*, 157(19), 2546–2557.

[13] Hall, J. W. (2006). Uncertainty-based sensitivity indices for imprecise probability distributions, *Reliability Engineering & System Safety*, 91(10–11), 1443–1451.

[14] Helton, J. C. and Oberkampf, W. L. (eds.) (2004). Special Issue on Alternative Representations of Epistemic Uncertainty, *Reliability Engineering and System Safety*, 85(1–3).

[15] Igusa, T., Buonopane, S. G., and Ellingwood, B. R. (2002). Bayesian analysis of uncertainty for structural engineering applications, *Structural Safety*, 24(2–4), 165–186.

[16] Klir, G.J. (2006). *Uncertainty and Information: Foundations of Generalized Information Theory*, Wiley-Interscience, Hoboken.

[17] Kozine, I. and Filimonov, Y. (2000). Imprecise reliabilities: experiences and advances, *Reliability Engineering and System Safety*, 67, 75–83.

[18] Kruse, R. and Meyer, K. (1987). *Statistics with Vague Data*, Reidel, Dordrecht.

[19] Li, S., Ogura, Y. and Kreinovich, V. (2002). *Limit Theorems and Applications of Set Valued and Fuzzy Valued Random Variables*, Kluwer Academic Publishers, Dordrecht.

[20] Melchers, R. E. (2003). Probabilistic Model for Marine Corrosion of Steel for Structural Reliability Assessment, *Journal of Structural Engineering*, 129(11), 1484–1493.

[21] Möller, B., Graf, W. and Beer, M. (2000). Fuzzy structural analysis using α-level optimization, *Computational Mechanics*, 26(6), 547–565.

[22] Möller, B., Graf, W. and Beer, M. (2003). Safety assessment of structures in view of fuzzy randomness, *Computers and Structures*, 81(15), 1567–1582.

[23] Papadimitriou, C., Beck, J. L. and Katafygiotis, L. S. (2001). Updating robust reliability structural test data, *Probabilistic Engineering Mechanics*, 16, 103–113.

[24] Schenk, C. A. and Schuëller, G. I. (2005). *Uncertainty Assessment of Large Finite Element Systems*, Springer, Berlin, Heidelberg.

[25] *USFOS, Ultimate Strength of Frame Offshore Structures 2001*, (2001). User's Manual, SINTEF.

[26] Utkin, L. V. (2004). An uncertainty model of structural reliability with imprecise parameters of probability distributions, *ZAMM - Zeitschrift für Angewandte Mathematik und Mechanik*, 84(10–11), 688–699.

[27] Viertl, R. (1996). *Statistical Methods for Non-Precise Data*, CRC Press, Boca Raton New York London Tokyo.

[28] Walley, P. (1991). *Statistical Reasoning with Imprecise Probabilities*, Chapman & Hall, London.

[29] Weichselberger, K. (2000). The theory of interval-probability as a unifying concept for uncertainty, *International Journal Approximate Reasoning*, 24(2–3), 149–170.

[30] Yamauchi, Y. and Mukaidono, M. (1999). Interval and paired probabilities for treating uncertain events, *IEICE Transactions on Information and Systems*, E82_D(5), 955–961.

[31] Zimmermann, H. J. (1992). *Fuzzy Set Theory and Its Applications*, Kluwer Academic Publishers, Boston London.