Dirichlet model *versus* expert knowledge.

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Abstract

Decision theory is used to choose a portfolio. Elicitation methods was used based on the utility function and from expert opinion thus, enabling the creation of a utility function for the investor and another for the a priori distribution on economic indicators. The model chosen for an investment portfolio was formulated based on decision theory, incorporating aspects of systematic and unsystematic risk. The model was developed so as to structure an efficient way to understand the application of decision theory in the financial market as well as the application of the Imprecise Dirichlet Model-IDM. The IDM allows the use of imprecise probability. Finally, the IDM was compared to the Markowitz method and also, to the decision model, using only expert opinion, considering an allocation over time to verify which of the three models was the best one. The final conclusion is that expert opinion should not be neglected in her compiling a portfolio.

Keywords. Linear Programming, Elicitation, Portfolio Selection, Financial.

1 Introduction

In the financial market, the portfolio selection problem consists of distributing the total amount available for investment among the financial "products" in the market. Hitherto, the Markowitz portfolio selection procedures, in [10], use ad hoc procedure. One of the numbers used most frequently as a guide, was the average value of the investment payback, usually estimated from past data. The Markowitz procedure is essentially a trade-off between the average and the standard deviation of the (future) payback. It is implemented as a quadratic programming problem: either one minimizes the standard deviation (risk, in the jargon) subject to the constraint that the average must be greater than some previously determined value (usually taken to be zero), or one which maximizes the average payback, subject to an upper bound constraint on the risk. This article suggests using decision theory in the portfolio selection problem. It is divided into five sections. Introduction sets the context and present of the other sections. The second is a brief review of articles related to portfolio selection and imprecise probability. The third presents a decision model that incorporates elements of the economy, such as indicators of economic scenarios that result in the compiling the portfolio. The fourth section presents methods to elicit the utility function and expert knowledge. the measures are used in comparison with the Imprecise Dirichlet Model – IDM. Finally, some conclusions are drawn from the main results.

According to [8] "*Developments in portfolio are stimulated by two basic requirements: (1) adequate modeling of utility functions, risks and constratints; (2) efficiency, i.e., ability to handle large numbers of instruments and scenarios.*" This paper presents a model that satisfies both conditions.

2 A Review of the Literature

Markets in which the price reflects the available information are called efficient markets. The idea of efficient markets is the premise for the Markowitz method. The estimated average return, $\overline{R(A)}$ and the estimated risk $\hat{\sigma}$ of an asset, are expressed by the mathematical expectation of past returns and its standard deviation. The equations below represent the estimate of the expected return and risk of an asset:

$$\overline{R(A)} = \frac{\sum_{t=1}^{n} R_t}{n} \tag{1}$$

$$\hat{\sigma}(A) = \frac{\sum_{t=1}^{n} (R_t - \overline{R(A)})^2}{n-1} \tag{2}$$

The number of observations is represented by n, and

R_t represents the return at time t.
Markowitz method is based on the formation of an asset portfolio so that the risk attributed to each asset can be minimized. This risk is called unsystematic risk. In the Markowitz method, the risk that is not being considered is the market risk, known as systematic risk. Markowitz idea consists is to diversify risk. Thus, the portfolio comprises assets with a negative correlation. Therefore, to the extent that one asset generates losses for the portfolio, another will generate earnings. The average return $R(P)$ and average risk $\sigma(P)$ of a portfolio are expressed by the following equations:

$$R(P) = \sum_{j=1}^{n} R_j W_j \tag{3}$$

$$\sigma(P) = \left[\sum_{i=1}^{n} \sum_{j=1}^{n} W_i W_j \rho_{i,j} \sigma_i \sigma_j \right]^{1/2} \tag{4}$$

where

- The percentage of investment in each asset is W_j;
- σ_j represents the risk of each asset;
- $\rho_{i,j}$ are the coefficients of correlation between the return of two assets.

To obtain the percentage of investment in each asset the nonlinear programming method is used, in which the variables of choice are: the percentages of application. The functional objective is the risk of the Portfolio and the restrictions are quite logical. Given that the percentage of implementation is a probability, it will be positive and the sum of the percentages will be equal to one. The problem is expressed as follows:

$$\min_{W_j} \sigma(P)$$

$$s.a$$

$$\sum W_j = 1 \ , \ W_j \geqslant 0$$

2.1 Probability in Finance Theory

An increasing number of studies are being developed in order to apply imprecise probability to *portfolio* models. At first, the models attempt to introduce the concept of *fuzziness* into the necessary measures for implementing Markowitz model. Examples of *fuzzy* being applied to the development of a *portfolio* are [13], [6] and [2]. Another application of imprecise probability in portfolio management is to seek conditions for separations of the investment fund. In [7] there is an introduction of classical conditions in order to divide funds, and in [12] there is an application subadditive probabilities, where the possibility of inertia in the choice of optimal portfolios is proved. The studies by applying imprecise probability to the economy, but the ideas are going in the direction of finding coherent risk measures and/or price arbitrage of assets.

3 The Decision Model

The model was proposed in [1], which used a simple characterization of the economic scenario, by reducing it to a unique economic indicator $\theta \in [0, 1]$. The observations x (time series data) were modeled in the same way as economic scenarios. For example, if four economic indicators were used, one of them would have 16 scenarios. These scenarios were ordered from worst to best, and an integer number was attached to each of them. The better the scenario, the larger the integer. So, the likelihood function is, for that model, a binomial distribution.

$$P(x|\theta) = \binom{n}{x} \theta^x (1-\theta)^{n-x}$$

The prior distribution of θ is the Beta distribution

$$\pi(\theta) = \frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\Gamma(\beta)} \theta^{\alpha-1} (1-\theta)^{\beta-1}.$$

3.1 The elements of the problem

The notation is as follows:

ξ_i = i^{th} financial product (i^{th}asset);

a_i = fraction of the available initial capital to be invested in asset ξ_i;

p = net return (payback) of the portfolio , $p \in [-M, M]$, $M > 0$;

GIP_t = gross internal product in period t;

IR_t = inflation rate in period t;

PR_t = prime rate in period t;

UN_t = Unemployment in period t;

The states of nature are defined as follows. First, one defines an intermediate variable ω_i:

Let X_t be an economic indicator, if X_{t+1} is better for the economy than X_t, one then writes $\omega_{t+1} = 1$;

otherwise, $\omega_{t+1} = 0$. Since there are four economic indicators (GIP_t,IR_t, PR_t, UR_t) there will be 16 economic scenarios for each period (one month). Table 1 shows the 16 scenarios which will constitute the states of nature in the decision theory model.

Table 1: The Possible 16 Scenarios.

Scenarios	ω_1	ω_2	ω_3	ω_4
θ_1	0	0	0	0
θ_2	0	0	0	1
θ_3	0	0	1	0
θ_4	0	0	1	1
θ_5	0	1	0	0
θ_6	0	1	0	1
θ_7	0	1	1	0
θ_8	0	1	1	1
θ_9	1	0	0	0
θ_{10}	1	0	0	1
θ_{11}	1	0	1	0
θ_{12}	1	0	1	1
θ_{13}	1	1	0	0
θ_{14}	1	1	0	1
θ_{15}	1	1	1	0
θ_{16}	1	1	1	1

So, $\Theta = \{\theta_1, \theta_2 \ldots \theta_{16}\}$.

Scenarios θ_1 and θ_{16} are the worst and the best, respectively, for economy. The remaining ones are not naturally orderable, since the effects they have in the economy will depend upon a series of other characteristics of the specific country. Thus, the θ_js are essentially categorical.

3.2 Data

A time series of the 100 months is available for each of the four economic indicators, as well as for the financial assets to be used in the portfolio. It was thus possible to establish the evolution of the scenarios. These observations, x_j, correspond to a sample of a multinomial probability distribution:

$$P(x|\theta) = \frac{n!}{\prod_{j=1}^{16}(x_j!)}\theta_j^{x_j}$$

Table 2 shows the number of times that each of the scenarios occurred.

3.3 Dirichlet Prior Distribution

To incorporate expert opinion in this model, it is natural to use the conjugate prior distribution of the multinomial, which is the Dirichlet prior. The Dirichlet prior density then is:

Table 2: scenarios occurring.

x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8
1	4	8	7	4	6	9	4
x_9	x_{10}	x_{11}	x_{12}	x_{13}	x_{14}	x_{15}	x_{16}
6	6	4	13	6	5	6	11

$$\gamma(\theta) = \frac{\Gamma(\nu)}{\prod_{j=1}^{16}\Gamma(\alpha_j)}\prod_{j=1}^{16}\theta_j^{\alpha_j-1}$$

where $\nu = \sum \alpha_j$, $\alpha_j > 0$, $\sum \theta_j = 1$.

The parametrization used in [15] will also be used here:

$$\pi(\theta) = \frac{\Gamma(\nu)}{\prod_{j=1}^{16}\Gamma(st_j)}\prod_{j=1}^{16}\theta_j^{st_j-1}$$

where $\nu = s\sum t_j$, $\sum \theta_j = 1$, $s > 0$; s is called a hyperparameter.

3.4 Dirichlet Posterior Distribution

When combined, by Bayes rule, with the multinomial likelihood function $P(x|\theta)$, the Dirichlet prior density generates density function a posteriori

$$\pi(\theta|x) = \frac{\Gamma(\upsilon)}{\prod_{j=1}^{k}\Gamma(\alpha_j + x_j)}\prod_{j=1}^{k}\theta_j^{\alpha_j-1+x_j},$$

where $\upsilon = \sum \alpha_j + x_j$. The set of all distributions a posteriori is defined by:

$$t^* = \frac{n_j + st_j}{N + s}. \tag{5}$$

3.5 The Action Space

The investment alternatives constitute the action space $A = \{a\}$. Each a is a mix of financial assets, and is a vector of nonnegative numbers that add up to one. The following assets were used:

- Bank Certified Deposit (CDB) (30 days rentability average);

- Gold - percentage of monthly variation;
- Ibovespa - Sao Paulo Stock Monthly Average Growth Rate;
- Financial Assets Fund (FAF) - Accumulated monthly rentability.

Table 3 shows a descriptive statistics of those assets, as well as the result of applying of the Markowitz portfolio (MP) selection procedure. In this procedure, the optimal action corresponds to CDB -0.9638, Gold - 0.0106, Ibovespa - 0.0049 and FAF - 0.0205.

The available series corresponds to the same period as those of the economic indicators, and were obtained from the Brazilian Central Bank.

Table 3: Descriptive Statistics.

Assets	**Mean**	**Min**	**Max**	**Std. Dev.**
CDB	2.06	1.15	5.20	0.91
GOLD	1.43	-16.40	70.00	8.95
IBOVESPA	1.73	-39.55	28.02	11.54
FAF	1.61	-17.98	17.07	5.67
MP	2.04	0.89	5.24	0.92

3.6 The Consequence Function

In [3], the choice of the analytical expression of the consequence function considers some aspects:

- States of nature and actions are merged in the right sense; θ and a work independently of each other, but they are merged to make up the probability distribution of p;
- It represents the behavior which is usually observed in the investment payback: unimodality, and bounded variance and asymmetry; some robustness is desirable, i.e., the persistence of a distribution's characteristic behavior under perturbations in the paraments;
- It should be analytically tractable when in association with the other analytical expressions the decision rule when calculating.

In the portfolio selection model [3] the following consequence function was suggested:

$$f(p|\theta,a) = [M(1+R(a))\theta(1-\theta)]^{-1} \text{ if } \tag{6}$$

$$M(1+R(a))\left[\frac{\theta}{2}(3+\theta)-1\right]+\mu(a) \leq p \text{ and}$$

$$p \leq M(1+R(a))\left[\frac{\theta}{2}(5-\theta)-1\right]+\mu(a);$$

$$f(p|\theta,a)=0, \text{ otherwise,}$$

where $a = [a_j]$ is the vector of fractions attributed to each asset; this corresponds to an action; $\mu(a)$ = average value of the portfolio, $R(a) = \left(1 - \frac{\mu(a)}{\mu(a)+\sigma(a)}\right)$ a measure of the risk of the portfolio. It is important to look at the consequence function (equation 6). A closer look will shed some light in the behavior of this function: the larger the value of θ, the better the economy. For $\theta = \frac{1}{2}$ one has:

$$\frac{\theta}{2}(3+\theta)-1 = -\frac{1}{8} \text{ and}$$

$$\frac{\theta}{2}(5-\theta)-1 = \frac{1}{8}.$$

If $\mu(a) = 0$ then one has a uniform distribution between $-(1/4)M$ and $(1/4)M$. For any portfolio, a has a uniform distribution between $-(1/8)M(1+R(a))+\mu(a)$ and $(1/8)M(1+R(a))+\mu(a)$

In this model, the generalization of the consequence function is:

$$f(p|\theta,a) = \left[2M(1+R)\tau\prod\theta_j\right]^{-1} \text{ if}$$

$$M(1+R)\left[\sum n_j\theta_j + \tau\prod\theta_j\right]+\mu \geq p \text{ and}$$

$$M(1+R)\left[\sum n_j\theta_j - \tau\prod\theta_j\right]+\mu \leq p \;\; ;$$

$$f(p|\theta,a)=0 \quad \text{otherwise,}$$

where τ is a proportionality constant and n_j represents the impact of each θ_j in the consequence function.

3.7 Loss Function

Consider the quadratic utility function:

$$v(p) = k_0 + k_1 p - k_2 p^2.$$

The loss function is denoted by $L(\theta,a)$. It is defined as:

$$L(\theta,a) = -k_0 - k_1\left[M(1+R)\left[\sum n_j\theta_j\right]+\mu\right]$$
$$+k_2\left[M(1+R)\left[\sum n_j\theta_j\right]+\mu\right]^2 +$$
$$+\frac{1}{3}k_2\left[M(1+R)\tau\prod\theta_j\right]^2$$

3.8 The Bayes risk

To apply of the Bayes rule the following calculations are necessary:

1. $$u(f(p|\theta,a_j)) = \int u(p)f(p|\theta,a_j)dp.$$

2. $$L(\theta,a_j) = -u(f(p|\theta,a_j)).$$

3. $$R_d(\theta) = \sum_x P(x|\theta)L(\theta,d(x)).$$

4. $$r_d = \int_0^1 \pi(\theta)R_d(\theta)d\theta \text{ (Bayes risk).}$$

5. $$r_d = \int_0^1\left[\sum_x \pi(\theta)P(x|\theta)L(\theta,d(x))\right]d\theta.$$

6. $$r_d = \int_0^1\left[\sum_x \pi(\theta|x)P(x)L(\theta,d(x))\right]d\theta.$$

7. $$rd = \sum_x P(x)\int_0^1 \pi(\theta|x)L(\theta,d(x))d\theta.$$

8. To minimize r_d by a choice of d, which is the same as to minimize, for each x, the term
$$\int_0^1 \pi(\theta|x)L(\theta,d(x))d\theta,$$
by a choice of $d(x)$.

To facilitate the calculations one denotes $\frac{\Gamma(\nu)}{\prod_{j=1}^{k}\Gamma(\alpha_j+x_j)} = \omega$

$$r_d = \int_0^1 -\omega\left[\prod_{i=1}^{k}\theta_i^{\alpha_i-1+x_i}\right]\times$$
$$\times[k_0 + k_1\left[M(1+R)\left[\sum n_j\theta_j\right]+\mu\right]$$
$$-k_2\left[M(1+R)\left[\sum n_j\theta_j\right]+\mu\right]^2\times$$
$$\times\frac{1}{3}k_2\left[M(1+R)\tau\prod\theta_j\right]^2 d\theta$$

$$\therefore r_d = -[k_0\omega\int_0^1\prod\theta_j^{\alpha_j-1+x_j}d\theta +$$

$$-\int_0^1 k_1[M(1+R)\sum n_j\theta_j+\mu]\omega\prod\theta_j^{\alpha_j+x_j-1}d\theta -$$

$$-\int_0^1 k_2[M(1+R)\sum n_j\theta_j+\mu]^2\omega\prod\theta^{\alpha_j-1+x_j}d\theta +$$

$$+\frac{1}{3}k_2\omega M^2(1+R)^2\int_0^1\tau\prod\theta_j^2\prod\theta_j^{\alpha_j-1+x_j}d\theta$$

One thus obtains the expression of the risk of adopting a decision rule:

$$rd = -\{k_0 + k_1M(1+R)\omega\times$$
$$\sum_{i\neq j}^{k}\left[\frac{n_j}{(\alpha_j+x_j+1)\Pi(\alpha_i+x_i)}\right]+$$
$$k_1\mu - k_2[M^2(1+R)^2\omega\times$$
$$\left[\sum_{i\neq j}^{k}\frac{n_j^2}{(\alpha_j+x_j+1)\Pi(\alpha_i+x_i)}\right]+$$
$$2\sum_{j=1;i<j}^{k}\frac{n_jn_i(\prod_{t:t\neq j\neq i}(\alpha_t+x_t))^{-1}}{(\alpha_j+x_j+1)(\alpha_i+x_i+1)}+$$
$$M(1+R)\omega\sum_{j=1\ ;j\neq i}^{k}\frac{n_j(\prod(\alpha_i+x_i))^{-1}}{(\alpha_j+x_j+1)}+\mu]+$$
$$\frac{1}{3}\tau\omega M^2(1+R)^2k_2\prod_{j=1}^{k}\frac{1}{(\alpha_j+x_j+1)}\}$$

4 The Expert *Versus* The IDM

4.1 Utility

The elicitation of the utility by the original method developed by Von Neumann and Morgenstern occurs when an individual responds to only one question about the likelihood such that he becomes such individual indifferent between a consequence, P, or about a game with a probability λ to win $\overline{P}$ or $(1-l\lambda)$ to get $\underline{P}$. The questions are put in the form game or lottery. Game layout can also vary depending on operational convenience to applied method.

An elicitation protocol (some questions) was applied to the individual in order for him to declare the value of λ for which he feels indifferent between a certain amount and a game (lottery). It should be noted that there is no "right answer" for each question. However, it is necessary to be careful about obtaining a good insight in order to obtain good accuracy. The answers are individual and must be tailored to the individual psychology of risk. There will never be perfect accuracy; one must not confuse rationality with perfection.

The assumption for use of a von Neumann-Morgenstern weak cardinal utility function is that these are two goods, one of them more desirable, $\overline{P}$, and the other one is less desirable, $\underline{P}$, which assigns two arbitrary utilities. When these values $\overline{P}$ and $\underline{P}$ are distant from each other, it is very difficult to choose the value of λ for a given value P, where $\underline{P} < P < \overline{P}$. Thus, we must ask what is the value of λ which makes P indifferent to a lottery between $\underline{P}$ and $\overline{P}$ in different overlapping limits. Later, as the utility function is an interval measure, λ values must be passed to the same. It is intended to elicit the utility function of money in a range from - R$ 95,000.00 (minus ninety-five thousand reais) to R$ 95,000.00 (ninety-five thousand reais). After the questions, a regression is used to infer the error of the the decision-maker when Like answered the questions. A quadratic function expression was used. In which the were parameters $k_0 = 0.7025$, $k_1 = 0.0047$, and $k_2 = 1.7608 \times 10^{-5}$, for one individual (an investor).

4.2 The Expert

Keynes, at the beginning of his book, *Treatise on Probability*, cites Leibniz, who is already tired of saying that there is a new logic that deals with degrees of probability. Keynes advocates the hypothesis that in the long term, we'll all be dead and that a historical series, that would make predictions about our future, would when Like answered exist. When there are few data or no data, the *a priori* knowledge of the expert should be used. A new elicitation procedure of *a priori* knowledge of the expert was presented in [5] and [11].

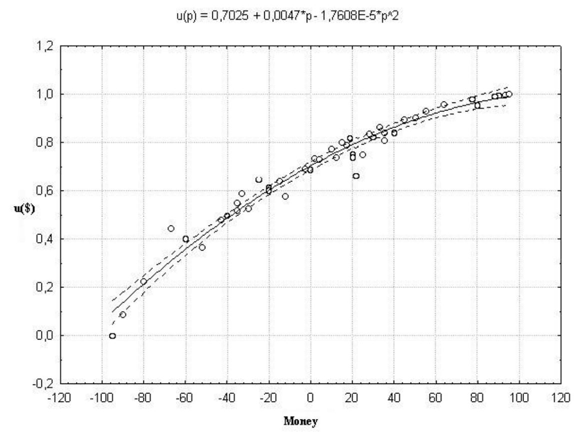

Figure 1: The Decision Maker's utility function.

The method used to elicit of the expert's prior distribution has the basic assumption that the expert has a vague knowledge about the state of nature, θ; It is assumed he can only make a finite amount of comparative probabilistic assertions when answering questions about the likelihood of the event belonging to one of two given ranges, I_A or I_B. For example, the expert will respond if it is more likely theta belongs to $I_A = [\theta_1, \theta_2, \theta_3]$ or $I - B = [\theta_4, \theta_5]$. This method expresses the expert's knowledge using a family of probability. Using this method allows, among other things, to make inferences about facts that cannot be presented by a historical series, but the facts there are and the probability of their occurrence is very high. These events change the decision on whether to invest or not in a financial asset. However, how significant is this change?

The model for solving two linear programming problems, mathematically, is expressed as follows: First of all, it is necessary to solve the maximization problem and later the minimization problem. Both are subject to the same set of constraints. These problems can be expressed as follows:

$$\max(\min)_{\theta_j} \sum_{j=1}^{2n} c_j \theta_j \tag{7}$$

subject to:

$$a_{ik} \sum_{j=i}^{k} \theta_j - a_{lm} \sum_{j=l}^{m} \theta_j \leq b_s \tag{8}$$

$$\alpha_j \theta_j \leq \theta_{j+1}, j = 1, 2, \ldots, 2n-1, \alpha > 0 \quad (9)$$

$$\beta_j \theta_{j+1} \leq \theta_j, j = 1, 2, \ldots, 2n-1, \beta > 0 \quad (10)$$

$$\theta_j \geq 0, j = 1, 2, \ldots, 2n \quad (11)$$

$$\sum_{j=1}^{2n} \theta_j = 1 \quad (12)$$

The values of c_j were randomly determined in order to allow a random search process. The restriction 8 becomes a questions in a questionnaire where the expert must answer them, and depending on the responses the signal may be $\leq$ or $\geq$.

Depending on the combination of parameters a_{ik}, a_{lm} e b_s, the expert's opinion can be collected in various ways. Constraints 9 and 10 are used when one wants to use a *a priori* distribution, so that such distribution may be as informative as possible. Otherwise, a good option is to suppress these restrictions. The remaining restrictions are considered the basic ones acceded to obtain a probability distribution. To obtain the probability distributions from an expert's opinion, he/she must be consistent in his/her responses. If a response is not consistent with all the other ones, the feasible set of restrictions will be empty. The expert must not answer these questions. The expert does not answer the questions when he/she cannot say anything about the fact of the likelihood of θ belonging or not belonging to one of the existing intervals. The questions which the expert does not answer will not enter the constraints of the linear programming problem. Questions will be displayed according to the indicators shown in [5]. The model defines new constructs such as vagueness, precision, concordance, overall vagueness, conflicts, decidability, harmony, quality of inference and amount of information. The elicitation method for linear programming also allows the combination of bodies of evidence.

After analysing a questionnaire referring to the 16 scenarios presented in Table 1 and solving the linear programming problem above, for different values of c_j, the result shown in Table 4 was obtained. This result can be interpreted as a convex set of probabilities within a range with an upper and lower probability for each state of nature. Any combination of values within the ranges can be used as a prior distribution of the expert.

Table 4: Expert opinion.

Scenarios	$\underline{\pi(\theta)}$	$\overline{\pi(\theta)}$
θ_1	0,00%	6,25%
θ_2	0,00%	1,67%
θ_3	5,00%	5,00%
θ_4	4,58%	5,00%
θ_5	1,67%	5,00%
θ_6	1,67%	3,33%
θ_7	3,33%	3,33%
θ_8	0,00%	2,50%
θ_9	3,33%	4,17%
θ_{10}	7,08%	8,33%
θ_{11}	10,83%	12,50%
θ_{12}	3,33%	6,67%
θ_{13}	5,83%	6,67%
θ_{14}	6,67%	7,50%
θ_{15}	7,50%	9,17%
θ_{16}	25,83%	26,25%

4.3 Imprecise Dirichlet Model

One of the hypotheses of the model is the existence of a prior distribution, $\pi(\theta)$. In [15] a way is presented for obtaining posterior distributions without having a prior distribution. This model is known as the imprecise Dirichlet model (IDM). From the set of posterior Dirichlet distributions, one obtains upper and lower probabilities for the event θ_j. The lower probability is obtained by making $t_j \to 0$ and the upper probability is obtained by making $t_j \to 1$ in Equation 5. One will then get:

$$\overline{P}(\theta_j|x) = \frac{n_j + s}{N + s}, \quad \text{and}$$

$$\underline{P}(\theta_j|x) = \frac{n_j}{N + s}.$$

where N is the number of observations about θ. In the example, $N = 100$. As discussed in [15], $s = 1$ corresponds to a frequentist outlook, and $s = 2$ to a cautious Bayesian. Table 5 shows the results that were obtained by the IDM in these two cases.

4.4 Comparisons

The comparison among the three forms of selecting a portfolio is considering the time in which information is available. The criterion of minimizing the maximum risk will be used; This result is obtained by calculating the lowest upper risk. Using the upper posterior distributions and the upper probabilities of the expert, the Monte Carlo method was used to calculate which is the action of lowest and highest risk. During

Table 5: Upper and Lower Probability.

s	$s = 1$		$s = 2$	
$P(\theta\|x)$	$\underline{P}$	$\overline{P}$	$\underline{P}$	$\overline{P}$
$P(\theta_1\|x)$	0.0099	0.0198	0.0098	0.0294
$P(\theta_2\|x)$	0.0396	0.0495	0.0392	0.0588
$P(\theta_3\|x)$	0.0792	0.0891	0.0784	0.0980
$P(\theta_4\|x)$	0.0693	0.0792	0.0686	0.0882
$P(\theta_5\|x)$	0.0396	0.0495	0.0392	0.0588
$P(\theta_6\|x)$	0.0594	0.0693	0.0588	0.0784
$P(\theta_7\|x)$	0.0891	0.0990	0.0882	0.1078
$P(\theta_8\|x)$	0.0396	0.0495	0.0392	0.0588
$P(\theta_9\|x)$	0.0594	0.0693	0.0588	0.0784
$P(\theta_{10}\|x)$	0.0594	0.0693	0.0588	0.0784
$P(\theta_{11}\|x)$	0.0396	0.0495	0.0392	0.0588
$P(\theta_{12}\|x)$	0.1287	0.1386	0.1275	0.1471
$P(\theta_{13}\|x)$	0.0594	0.0693	0.0588	0.0784
$P(\theta_{14}\|x)$	0.0495	0.0594	0.0490	0.0686
$P(\theta_{15}\|x)$	0.0594	0.0693	0.0588	0.0784
$P(\theta_{16}\|x)$	0.1089	0.1188	0.1078	0.1275

the data series, the portfolio with lowest upper risk was calculated while the information was obtained. The return that an investor would obtain over the 100 months by using the Markowitz method for compiling a portfolio was calculated as well. The cumulative return by the IDM during the period analyzed was the following: for $s = 1$ it was 557.71%, for $s = 2$, it was 519%. If the investor had used the Markowitz method the cumulative return would be 754.98%. The return would have been 820 % if the expert's opinion had been used.

5 Conclusions

An interesting point regarding the model is that the formulation is general, broad and flexible. Thus, there is the option of using other analytical expressions. Another more general observation is that the better the economic theory being used in the preparation of constructs, the better the results should be. The main conclusions of this article are the following:

- Subjective aspects can be used such as: the utility of the investor and expert's opinion can be measured and used to guide the decision-making in the financial markets;

- The expert's opinion about uncertain states of the world can be used as a measure of systematic risk. Thus, uncertainty about events like the presidential election, agreements and international wars are measured and incorporated into the problem of choosing the investment;

- The imprecise Dirichlet model presents an important advanced in making the decisions with insufficient information. Besides, this model should be used in problems involving the choice of investment portfolios. It is possible to incorporate the expert's opinion. Moreover, information from these bodies of evidence should be used together, since the result of the application shows that it is not correct to disregard the expert opinion;

- The use of analytical models can lead to theoretical conclusions about the investor's behavior;

- Analytical models are also easy to implement: they can be used in a spreadsheet or a calculator.

The Consequence Function is perfect for the implementation of models based on Condicional Value-at-Risk (CVAR) [14] and [8]. Comparisons between the model presented in this article and CVAR are objects of future works.

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