# Likelihood-based Naive Credal Classifier

## A movie (paper) by

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# THE CAST

## THE BAYESIAN

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**BIO** @IDSIA,
PhD in Engineering,
Bayesian/credal
classification

## THE AGNOSTIC

Alessandro Antonucci

**BIO** @IDSIA,
PhD Comp Science,
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## THE FREQUENTIST

Marco Cattaneo

**BIO** @LMU,
PhD Statistics,
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# EPISODE I

# “Crises of Faith”

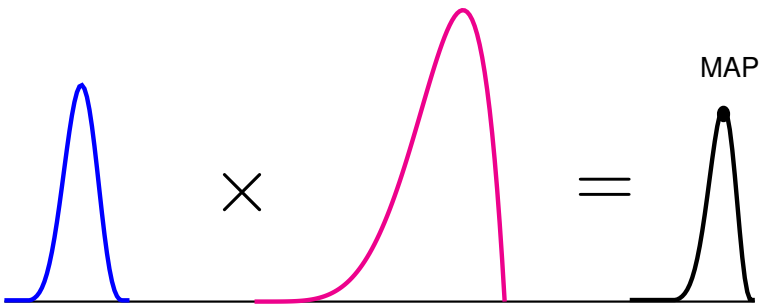
## Learning probabilistic models from data

Learning from data $\mathcal{D}$ (about random var $C$, $c \in \mathcal{C}$)

multinomial likelihood $P(\mathcal{D}) = \prod_{c \in \mathcal{C}} \theta_c^{n(c)}$ (counts $n$)

## BAYESIAN

prior $\times$ likelihood = posterior




priors $\times$ likelihood = posteriors

IDM prior ignorance with $Dir(s\mathbf{t})$

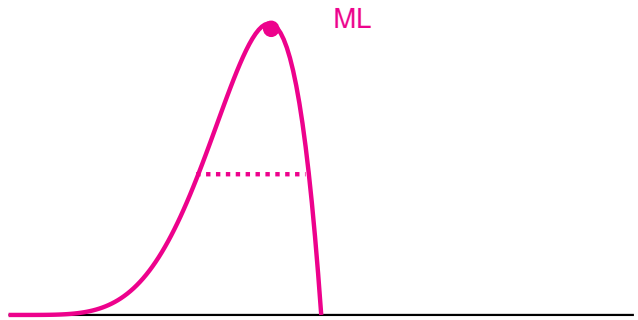
$\mathbf{t} \in \mathcal{T} := \{\mathbf{t} \,|\, \sum_{c \in \mathcal{C}} t(c) = 1, t(c) > 0\}$

$s$ equivalent sample size

($s = 0$ precise, $s \to \infty$ vacuous)

## FREQUENTIST

likelihood (only)




models with likelihood $\geq$ threshold

LIK refine starting credal set $\mathbf{P}$

$\mathbf{P}_\alpha := \{P \in \mathbf{P} \,|\, P(\mathcal{D}) \geq \alpha P_{ML}(\mathcal{D})\}$

threshold $\alpha \in [0, 1]$ ($\mathbf{P}_\alpha \subseteq \mathbf{P}$)

($\mathbf{P}_{\alpha=0} \equiv \mathbf{P}$, $\mathbf{P}_{\alpha=1} = P_{ML}$)

# EPISODE II

# “Becoming adults”

## Credal classification

# Probabilistic Classifiers

Class $C$, Features $\mathbf{F} := (F_1, \ldots, F_m)$, $f_i \in \mathcal{F}_i$

complete data $\mathcal{D} := \{(c^{(j)}, \mathbf{f}^{(j)})\}_{j=1}^{d}$

Which class label assign to instance $\mathbf{F} = \tilde{\mathbf{f}}$ ?

Precise classifiers learn joint $P(C, \mathbf{F})$

Assign to $\tilde{\mathbf{f}}$ most probable class label

$$\arg\max_{c' \in \mathcal{C}} P(c', \tilde{\mathbf{f}})$$

this is a classifier: $(\mathcal{F}_1 \times \ldots \times \mathcal{F}_m) \to \mathcal{C}$

Credal classifiers learn joint credal set $\mathbf{P}(C, \mathbf{F})$

Set of optimal class labels (e.g., maximality)

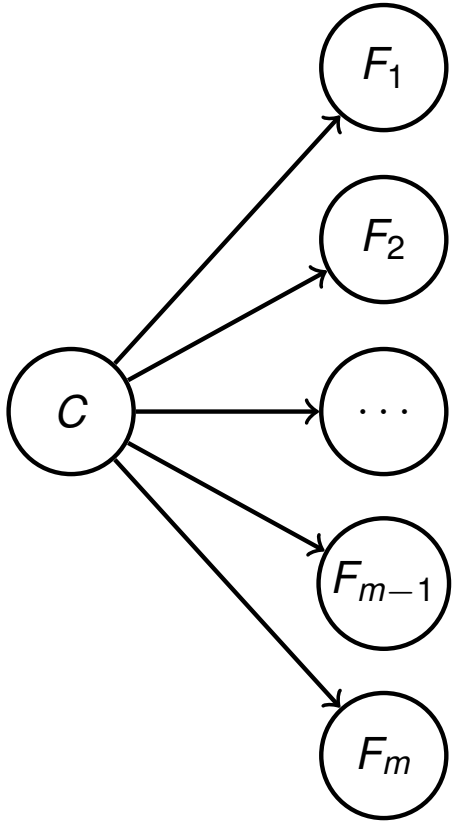
$$\{c' \in \mathcal{C} | \nexists c'' \in \mathcal{C} : P(c'', \tilde{\mathbf{f}}) > P(c', \tilde{\mathbf{f}}), \forall P \in \mathbf{P}\}$$

A credal classifier: $(\mathcal{F}_1 \times \ldots \times \mathcal{F}_m) \to 2^{\mathcal{C}}$

*(may) return multiple classes*

naive assumption

*"given the class, features are independent"*



# Naive Credal Classifiers (NCC)

BAYESIAN

- IDM-based NCC (Zaffalon, 2001)
- Efficient classification algorithm based optimization under the linear constraints

FREQUENTIST

- LIK-based NCC (this paper)
- Efficient classification algorithm based on analytical derivation of the likelihood upper envelope ($\alpha$-cuts identified numerically)

**Feature problem** zero joint counts $n(C = c', F_i = f_i) = 0$ make the classifier widely imprecisei

**NCC**$_\epsilon$ (Corani & Benavoli, 2010)

- Shrink the IDM set of priors by linear-vacuous contamination
- NCC$_{\epsilon=0}$ =NBC, NCC$_{\epsilon=1}$ vacuous

**NCC**$_\alpha$ (this paper)

- "semi-supervised" learning $\mathcal{D} := \mathcal{D} \cup (C = *, \mathbf{F} = \tilde{\mathbf{f}})$
- Assume $C$ missing-at-random for the incomplete instance $\tilde{\mathbf{f}}$

# EPISODE III

# “The final duel”

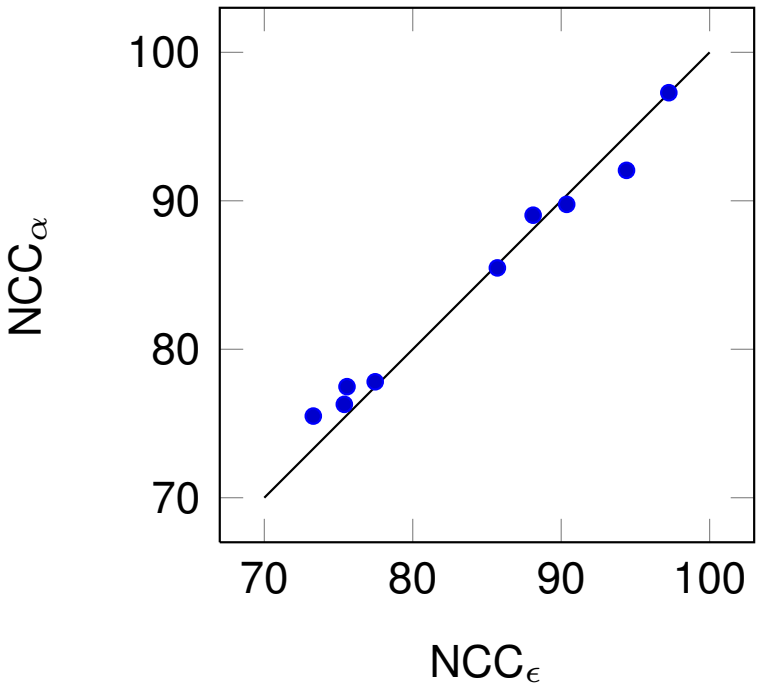
Comparing the two classifiers
(assuming you know how to deal with it)

Extensive tests on UCI datasets

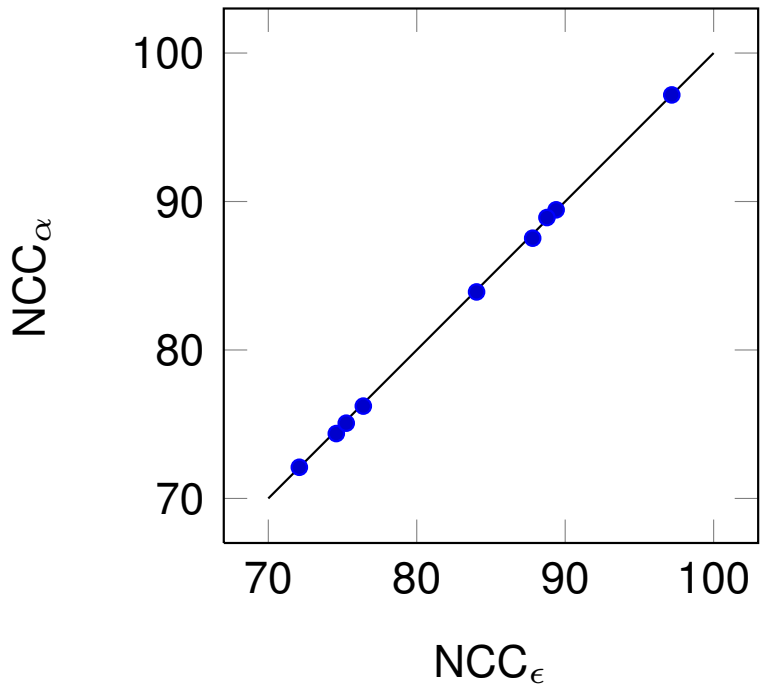
with JNCC2 (`idsia.ch/~giorgio/jncc2.html`)

$(\alpha, \epsilon)$ tuned to reach same average output size

Accuracy when precise




Overall accuracy (discounted)




Very similar performances, according to the available performance descriptors

THE END
(ALMOST)

# What about the agnostic?

- The agnostic is still agnostic
- None of the two classifiers clearly outperforms the other (according to the actual metrics)
- “Bayesian” approach has a clear behavioural interpretation
- “Frequentist” approach promising for analytical results even with more complex independence structures
- If no classifier outperforms the other, use them sequentially !
  Future work: indecision in $\text{NCC}_\epsilon$ could be resolved by $\text{NCC}_\alpha$ minimum $\alpha$ resolving indecision as a confidence level

# THE END

*All characters appearing in this work are fictitious. Any resemblance to real persons, living or dead, is purely coincidental.*