Robust detection of exotic infectious diseases in animal herds

A comparative study of two decision methodologies under severe uncertainty

Matthias C. M. Troffaes[1] John Paul Gosling[2]

[1]Durham University, United Kingdom

[2]John Paul Gosling, The Food and Environment Research Agency (FERA), United Kingdom

25 July, 2011

About Us

- Matthias Troffaes
  - Gent → Pittsburgh → Durham
  - decision making under severe uncertainty
  - computational tools
  - algorithms
  - engineering applications
- John Paul Gosling
  - York → Leeds
  - elicitation
  - modelling of food consumption and exposure to contaminants
  - Bayes linear methods
  - uncertainty and sensitivity analysis of complex computer models
  - risk assessments

Inspection Protocols

- animals are transported and pass through customs
- some of them may have dangerous infectious diseases
- how many animals to test, yet avoid cataclysmic events?

Model

extension of Moffitt [2]

Key Features

- allow for imperfect testing
- cost term for terminating the herd
- cost term for apocalypse
- diseased animals modelled as binomial process
- worst-case assumption of independence between animals

Main Quantities of Interest

- r: uncertain probability of a single animal being viciously infected
- m: number of animals to test (to be decided)
- $L(m|r)$: expected loss for testing m animals, given r

Model

$$L(m|r) = \sum_{d=0}^{n} [c(m) + t(n)\Pr(T|d,m) + a(d)\Pr(T^c|d,m)] \times \binom{n}{d} r^d (1-r)^{n-d}$$

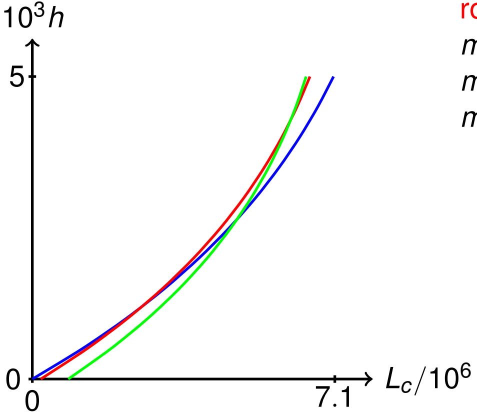
with

$$\Pr(T^c|d,m) = \sum_{z=0}^{d} (1-p)^z q^{m-z} \frac{\binom{d}{z}\binom{n-d}{m-z}}{\binom{n}{m}}$$

Info-Gap Analysis [1]

select decision which meets a given performance criterion L_c under the largest possible range of r

$$\hat{h}(m, L_c) = \max_{h \geq 0} \left\{ h \colon \max_{r \in [0,h]} L(m|r) \leq L_c \right\}$$



robustness curves

$m = 1$ (blue)

$m = 15$ (red)

$m = 30$ (green)

$L_c/10^6$	m^*	$10^3 \hat{h}(m^*, L_c)$
0.5	2	0.207
1.5	5	0.661
2.5	8	1.184
3.5	11	1.803

Maximality [3]

- horizon h via vacuous upper prevision

$$\overline{P}_h(f) = \max_{r \in [0,h]} f(r)$$

- pick one of the undominated options, i.e. any m for which

$$\min_{m' \in \{0,1,\dots,n\}} \overline{P}_h \left[L(m'|r) - L(m|r) \right] \geq 0$$

	$10^3 h$			
m	0.207	0.661	1.184	1.803
0	−0.9	−0.9	−0.9	−0.9
1	1.1	1.1	1.1	1.1
2	1.4	3.1	3.1	3.1
3	−0.6	4.9	5.1	5.1
4	−3.1	2.9	7.1	7.1
5	−7.7	0.9	7.0	9.1
6	−14.3	−1.1	5.0	11.1
7	−22.9	−4.3	2.9	9.9
8	−33.4	−9.5	0.9	7.9
9	−46.0	−16.6	−1.1	5.8
10	−60.6	−25.9	−4.3	3.7
11	−77.2	−37.1	−9.5	1.7
12	−95.8	−50.3	−16.8	−0.4
13	−116.4	−65.6	−26.1	−2.9
14	−139.1	−82.9	−37.4	−7.4
15	−163.7	−102.2	−50.8	−14.1

Discussion

- info-gap and maximality give essentially the same result
- this is not a coincidence!

Info-Gap–Maximin Theorem

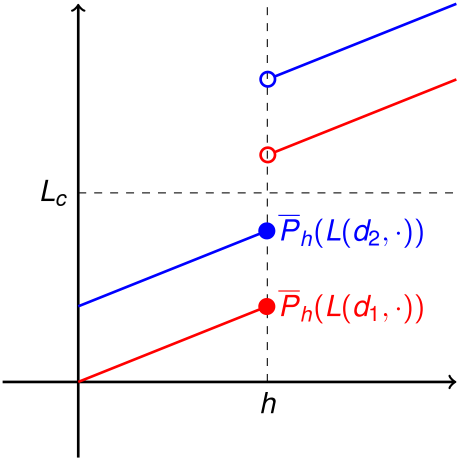
The info-gap solution $D^*(L_c)$ coincides with Γ-minimax solution with respect to $\overline{P}_h$ whenever the following conditions are satisfied:

1. for all $d \in D$, $\overline{P}_h(L(d, \cdot))$ is strictly increasing as a function of h
2. it holds that $L_c = \min_{d \in D} \overline{P}_h(L(d, \cdot))$.
   ('free' to choose L_c under the additional assumption of continuity)

Info-Gap–Maximin Theorem

Counterexample

What if there is no L_c such that $L_c = \min_{d \in D} \overline{P}_h(L(d, \cdot))$?



Info-Gap–Maximality Theorem

Let $L_c(h) = \min_{d \in D} \overline{P}_h(L(d, \cdot))$. Then, for all $h' \leq h$
every info-gap decision $d^* \in D^*(L_c(h'))$ is maximal with respect to $\overline{P}_h$.

Conclusion

- theoretical link between info-gap theory, Γ-minimax, maximality
- every info-gap solution is maximal
- info-gap analysis reveals (at least partly) set of maximal elements
- robustness curves also useful in imprecise probability context

Want to learn more?
Visit the poster!

Thank you!

References I

- Y. Ben-Haim.
  Information Gap Decision Theory: Decisions Under Severe Uncertainty.
  Academic Press, 2001.
- L. J. Moffitt, J. K. Stranlund, and C. D. Osteen.
  Robust detection protocols for uncertain introductions of invasive species.
  Journal of Environmental Management, 89:293–299, 2008.
- M. C. M. Troffaes.
  Decision making under uncertainty using imprecise probabilities.
  International Journal of Approximate Reasoning, 45(1):17–29, May 2007.