

A Verified MATLAB Implementation of Markov Set-Chains

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FIRST VERIFIED MSC IMPLEMENTATION

Aim: Integrate interval arithmetic features into Markov Set-Chains

- ◆ MATLAB
- ◆ INTLAB: Interval library

Define MSC

```

1 % Define transition matrix M=[P,Q]
2 P = [0, .25, .25; .25, .5, .25; .25, .25, 0];
3 Q = [0, .75, .75; .25, .5, .25; .75, .75, 0];
4 % Define initial vector x=[p,q]
5 p = [.1, .1, .2];
6 q = [.5, .6, .8];
7 % Create MSC object
8 Chain = MSC(P,Q,p,q);

```

- Uncertain parameters in
 - ◆ Transition matrices
 - ◆ Initial vectors
- Handling of round-off errors

Work with MSCs

```

1 % Get coefficient of ergodicity
2 Chain.CoeffOfErgodicity;
3 % Get HiLo-vector of M^2
4 [L,H]=Chain.getLH;
5 % Get bounds of L and H for each dimension
6 lb = getminmax(L);
7 ub = getminmax(H);
8 % Get verified bounds of x*M^5
9 [l5,h5]=Chain.getSMk(5);
10 % Get verified bounds of x*M^10
11 [l10,h10]=Chain.getSMk(10);
12 % Get verified bounds of x*M^20
13 [l20,h20]=Chain.getSMk(20);

```

- Verified Computation of
 - ◆ HiLo - method
 - ◆ Coefficient of ergodicity
 - ◆ Vertices

Plot Solution Spaces

```

1 % Plot solution space of l5 and h5
2 Chain.plot(l5,h5,'Solution space of $l_5$ and $h_5$');
3 % Plot solution space of l10 and h10
4 Chain.plot(l10,h10,'Solution space of $l_{10}$ and $h_{10}$');
5 % Plot solution space of l20 and h20
6 Chain.plot(l20,h20,'Solution space of $l_{20}$ and $h_{20}$');
7

```

Example of ergodic MSC

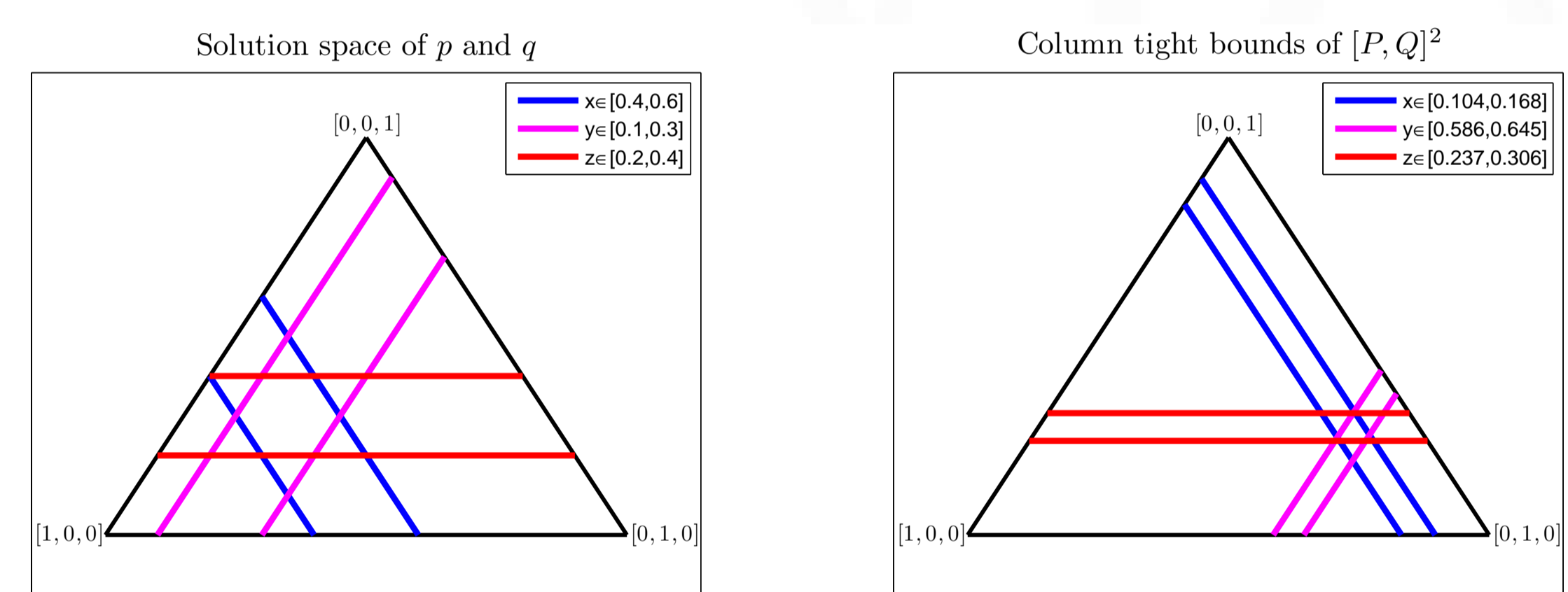
Goal: Compute Limit Solution Space



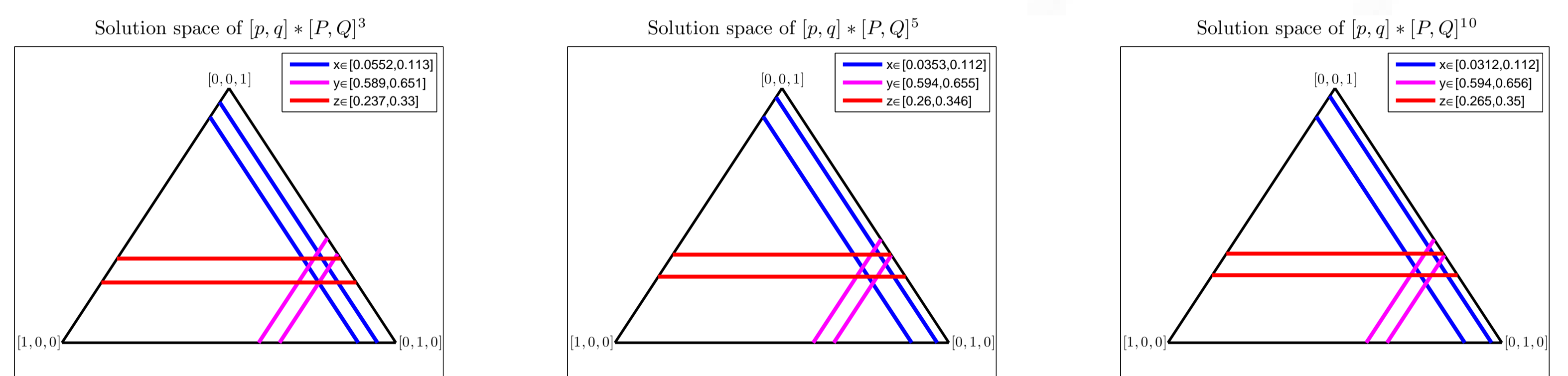
$$P = \begin{matrix} & A & B & C \\ A & .423 & .459 & .043 \\ B & .029 & .677 & .222 \\ C & 0 & .478 & .461 \end{matrix}$$

$$Q = \begin{matrix} & A & B & C \\ A & .473 & .509 & .093 \\ B & .079 & .724 & .272 \\ C & .036 & .528 & .511 \end{matrix}$$

$$p = (.4 \quad .1 \quad .2) \quad q = (.6 \quad .3 \quad .4)$$



Column tight bounds of step 3, 5 and 10



Advantages of MSC are:

- ◆ Verified results
- ◆ Unified treatment of uncertainty
- ◆ Fast function evaluation

Future work:

Application of verified MSC and fault tree analysis to modeling processes in SO fuel cells.