Modelling Common-Cause Failures Under Severe Prior Uncertainty

Matthias C. M. Troffaes[1] Dana Kelly[2]

[1]Durham University, Department of Mathematical Sciences, UK

[2]Idaho National Laboratory (INL), Nuclear Risk and Reliability Group, US

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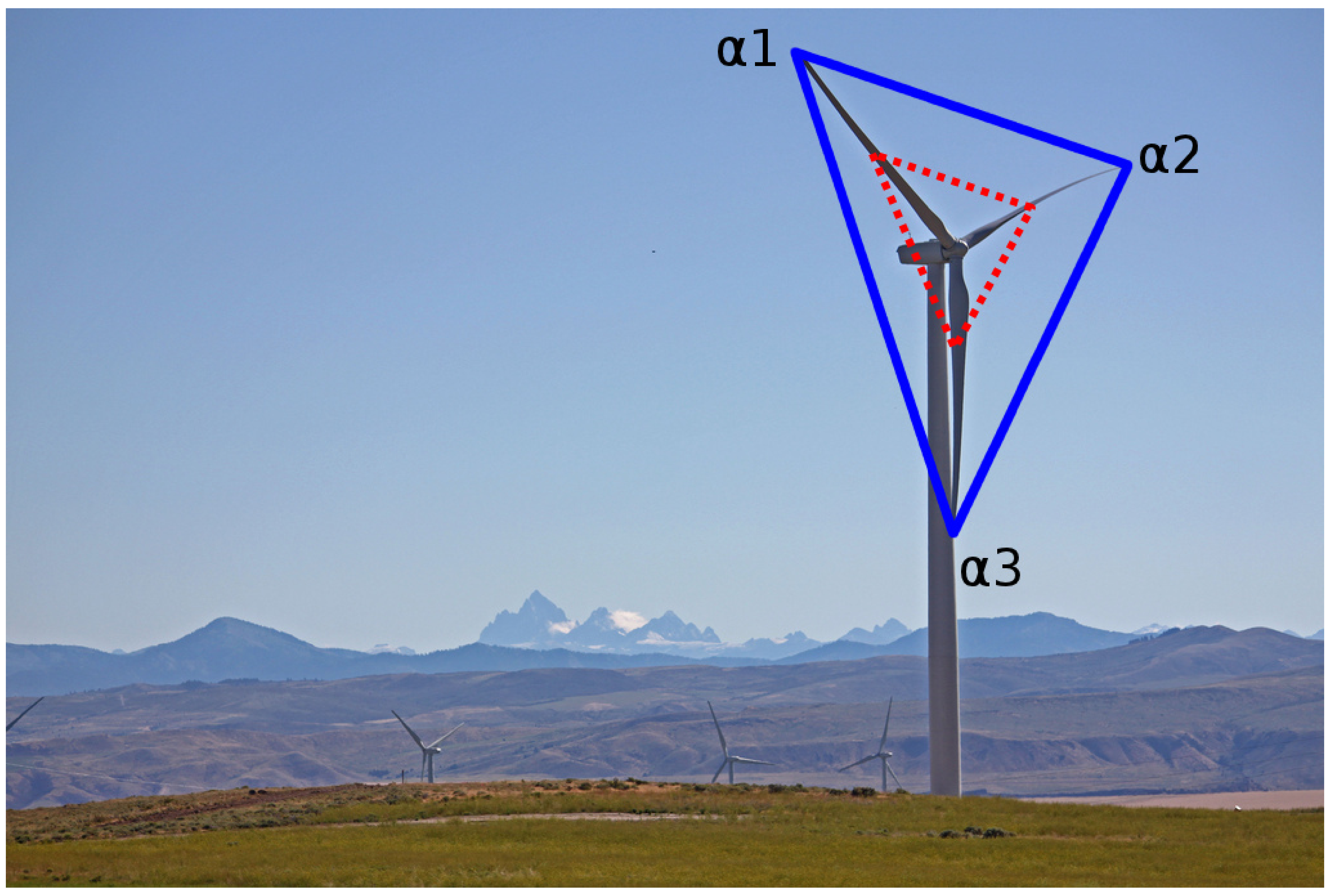


Example, Model, and Issues

Example

- adapted from [1]
- three components
- α_j: unknown probability of exactly j failed components (conditional on at least one failed component)
- n_j: data, counts cases with exactly j failed components

$\underline{P}(\alpha_1) = 0.950$	$n_1 = 35$
$\overline{P}(\alpha_2) = 0.030$	$n_2 = 1$
$\overline{P}(\alpha_3) = 0.015$	$n_3 = 0$
$\overline{P}(\alpha_4) = 0.005$	$n_4 = 0$

Alpha Factor Model

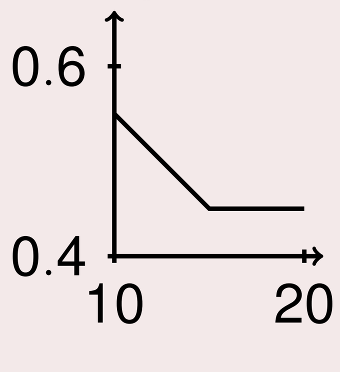
- developed for common cause failures [2]
- multinomial model + conjugate Dirichlet prior
- easily extendable to imprecise case

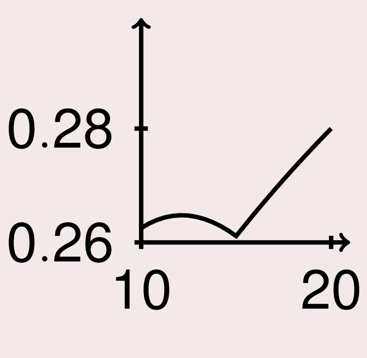
How To Pick The Prior?

- posterior sensitive to choice in non-informative prior
- constrained non-informative methods have too light tails
- main problem caused by (close to) zero counts

Going Imprecise: Arbitrary Set of Dirichlet Priors?

- 'supposedly' simple [4, p. 32, §6]—it's not!
- non-linear non-convex optimisation problems even if prior convex
  example (left = prior, right = posterior):





- elicitation difficult, not to say impossible
- need for simpler model without too much sacrifice in precision

Main Results

A Simple Generalised IDM Model

- prior set: $\{(s, \vec{t}) \colon s \in [\underline{s}, \overline{s}],\ t_j \in [\underline{t}_j, \overline{t}_j]\}$
  $0 \leq \underline{s} \leq \overline{s}$
  $\underline{t}_j$, $\overline{t}_j$: coherent lower and upper probability mass functions
- still non-linear, but convex
- easy to bound an arbitrary set by it
- generalises Walley's general beta-binomial model [3, p. 224, §5.4.3] to multinomial case
- elicitation is much more straightforward

Elicitation Of $\underline{t}_j$ And $\overline{t}_j$

prior lower and upper probabilities of exactly j components failing

Elicitation Of $\underline{s}$ And $\overline{s}$

- $\underline{s} = \overline{s} = 2$ is usually a horrible choice
- zero counts have too much influence on the posterior
  higher value of s needed to increase weight of prior
- but data *could be* right even for low counts
  prior-data conflict! $\rightarrow$ lower value of s to cover also data

general guideline:

- $\overline{s}$ is the number of one-component failures required to reduce the upper probability $\overline{t}_j$ ($j \geq 2$) of multi-component failure by half
- $\underline{s}$ is the number of multi-component failures required to reduce the lower probability $\underline{t}_1$ of one-component failure by half

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