Linear programming under vacuous and possibilistic uncertainty

Keivan Shariatmadar, Erik Quaeghebeur & Gert de Cooman
SYSTeMS Research Group, Ghent University
{Keivan.Shariatmadar,Erik.Quaeghebeur,Gert.deCooman}@UGent.be

Abstract

Consider the following (standard) linear programming problem: maximise a real-valued linear function $C^T x$ defined for optimisation variables x in $\mathbb{R}^n$ that have to satisfy the constraints $Ax \leq B,\ x \geq 0$, where the matrices A, B, and C are independent random variables that take values a, b, and c in $\mathbb{R}^{m\times n}$, $\mathbb{R}^m$ and $\mathbb{R}^n$, respectively. Using an approach we developed in previous work [3], the problem is first reduced to a constrained optimisation problem (co-problem) from which the uncertainties present in the description of the constraint are eliminated. The goal is to derive efficient solution techniques for this resulting co-problem.

We investigate what results can be obtained for two types of uncertainty models for the random variables A, B, and C – vacuous previsions and possibility distributions [see, e.g., 1, 5] – and for two different optimality criteria – maximinity and maximality [see, e.g., 4]. In our poster, we will present the problem description and show illustrated solutions for the most interesting cases we have investigated. We consider three variants of our problem: (i) when there is no uncertainty about C (this exactly fits the approach in [3]), (ii) when there is no uncertainty about B, which reduces to variant (i) when considering the dual, and (iii) the general case, which we can convert to the following problem: maximise the real value λ such that $Ax \leq B$, $C^T x \geq \lambda$ and $x \geq 0$, and which is the subject of current research. We here focus on variant (i).

For the different cases we studied, the co-problem and solution techniques derived are:

- Vacuous model relative to a set $\mathscr{A} \subseteq \mathbb{R}^{m\times n} \times \mathbb{R}^m$:
  - The maximin solution x_m can be found by solving the linear programming problem $\operatorname{argmax}_{x\in\mathbb{I}} c^T x$, where $\mathbb{I} := \bigcap_{(a,b)\in\mathscr{A}} \{x \in \mathbb{R}^n : ax \leq b\}$ is the inner feasibility space.
  - The maximal solutions can be found by vertex enumeration [see, e.g., 2] of $\left\{x \in \mathbb{R}^n : x \in \mathbb{O} \text{ and } c^T x \geq c^T x_m\right\}$, where $\mathbb{O} := \bigcup_{(a,b)\in\mathscr{A}} \{x \in \mathbb{R}^n : ax \leq b\}$ is the outer feasibility space, which turns out to be convex.
- Possibility distribution π on $\mathbb{R}^{m\times n} \times \mathbb{R}^m$ with corresponding lower probability $\underline{P}_\pi$:
  - The maximin solution is given by $\operatorname{argmax}_{x\in\mathbb{R}^n} \left(c^T x - L\right) \underline{P}_\pi(Ax \leq B)$ where L is a penalty for violating the constraints. When the possibility distribution π is unimodal then $\left(c^T x - L\right) \underline{P}_\pi(Ax \leq B)$ is unimodal too because of the linearity of the objective function, which allows us to find the maximin solution using a bisection method in which each step a linear programming problem must be solved.
  - We have not yet found an efficient way to calculate the maximal solutions. We can approximate the solutions when m and n are small enough.

Keywords. linear programming, maximinity, maximality, vacuous prevision, possibility distribution.

References

[1] D. Dubois and H. Prade. *Théorie des possibilités*. Masson, 2nd edition, 1988.
[2] B. Grunbaum. *Convex Polytopes*. Interscience Publishers, 1967.
[3] E. Quaeghebeur, K. Shariatmadar, and G. de Cooman. A constrained optimization problem under uncertainty. In *Proc. of FLINS 2010*, pages 791–796, 2010. doi:10.1142/9789814324700_0120. URL http://hdl.handle.net/1854/LU-973379.
[4] M. C. M. Troffaes. Decision making under uncertainty using imprecise probabilities. *Int. J. Approx. Reason.*, 45:17–29, 2007. doi:10.1016/j.ijar.2006.06.001.
[5] P. Walley. *Statistical Reasoning with Imprecise Probabilities*. Chapman and Hall, London, 1991.