Geometric conditional belief functions in the belief space

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Abstract

The original proposal for a conditioning operator in Dempster-Shafer's theory of belief functions (b.f.s) is due to Dempster himself [4], who formulated it in his original model in which belief functions are induced by multi-valued mappings $\Gamma : \Omega \to 2^\Theta$ of probability distributions defined on a set Ω onto the power set of another set ("frame") Θ. However, Dempster's conditioning was almost immediately and strongly criticized from a Bayesian point of view. In response to these objections a number of approaches to conditioning in the framework of belief functions (b.f.s) have been proposed along the years, in different mathematical setups.

Quite recently, the idea of formulating the problem geometrically has emerged. Lehrer [5], in particular, proposed such a geometric approach to determine the conditional expectation of non-additive probabilities (such as belief functions). The notion of generating conditional belief functions by minimizing a suitable distance function between the original b.f. b and the "conditioning region" $\mathcal{B}_A$, i.e., the set of belief functions whose basic belief assignment attaches mass to subsets of A only

$$b_d(.|A) = \arg\min_{b' \in \mathcal{B}_A} d(b, b') \tag{1}$$

has a clear potential. It expands our arsenal of possible approaches to the problem, and is a promising candidate to the role of general framework for conditioning. Most interestingly, Jousselme et al [1] have conducted a very nice survey of the distance or similarity measures so far introduced between belief functions, come out with an interesting classification, and proposed a number of generalizations of known measures. Many of these measures could be in principle plugged in the above minimization problem (1) to define conditional belief functions. In [3] the author computed all the conditional belief functions generated via minimization of L_p norms in the "mass space", where b.f.s are represented by the vector of their basic probabilities.

In this poster we explore geometric conditioning in the *belief space* $\mathcal{B}$, in which belief functions are represented by the vectors of their belief values $b(A)$. We adopt once again distance measures d of the classical L_p family, as a further step towards a complete analysis of the geometric approach to conditioning. We show that geometric conditional b.f.s in $\mathcal{B}$ are more complex than in the mass space, less naive objects whose interpretation in terms of degrees of belief is however less natural.

Keywords. Belief functions, conditioning, geometric approach, belief space, Lp norms.

References

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